Phase transition in gauge theories, monopoles and the Multiple Point Principle

C.R. Das[1], **L.V. Laperashvili**[1, 2]

[1] *The Institute of Mathematical Sciences, CIT Campus, Taramani, Chennai 600113, India*

[2] *Institute of Theoretical and Experimental Physics, B. Cheremushkinskaya 25, 117218 Moscow, Russia*

PACS: 11.15.Ha; 12.38.Aw; 12.38.Ge; 14.80.Hv

Keywords: gauge theory, duality, action, regularization, loop variables, strings

[1] **E–mail: crdas@imsc.res.in**
[2] **E–mail: laper@itep.ru**

Abstract

This review is devoted to the Multiple Point Principle (MPP), according to which several vacuum states with the same energy density exist in Nature. The MPP is implemented to the Standard Model (SM), Family replicated gauge group model (FRGGM) and phase transitions in gauge theories with/without monopoles. Using renormalization group equations for the SM, the effective potential in the two–loop approximation is investigated, and the existence of its postulated second minimum at the fundamental scale is confirmed. Phase transitions in the lattice gauge theories are reviewed. The lattice results for critical coupling constants are compared with those of the Higgs monopole model, in which the lattice artifact monopoles are replaced by the point-like Higgs scalar particles with magnetic charge. Considering our $(3+1)$–dimensional space–time as, in some way, discrete or imagining it as a lattice with a parameter $a = \lambda_P$, where λ_P is the Planck length, we have investigated the additional contributions of monopoles to the β–functions of renormalization group equations for running fine structure constants $\alpha_i(\mu)$ ($i = 1, 2, 3$ correspond to the $U(1)$, $SU(2)$ and $SU(3)$ gauge groups of the SM) in the FRGGM extended beyond the SM at high energies. It is shown that monopoles have N_{fam} times smaller magnetic charge in the FRGGM than in the SM (N_{fam} is a number of families in the FRGGM). We have estimated also the enlargement of a number of fermions in the FRGGM leading to the suppression of the asymptotic freedom in the non–Abelian theory. We have reviewed that, in contrast to the case of the Anti–grand unified theory (AGUT), there exists a possibility of unification of all gauge interactions (including gravity) near the Planck scale due to monopoles. The possibility of the $[SU(5)]^3$ or $[SO(10)]^3$ unification at the GUT–scale $\sim 10^{18}$ GeV is briefly considered.

Contents

1. Introduction: cosmological constant and the Multiple Point Principle

The contemporary low-energy physics of the electroweak and strong interactions is described by the Standard Model (SM) which unifies the Glashow–Salam–Weinberg electroweak theory with QCD — the theory of strong interactions.

The gauge symmetry group in the SM is :

$$SMG = SU(3)_c \times SU(2)_L \times U(1)_Y, \tag{1}$$

which describes the present elementary particle physics up to the scale ≈ 100 GeV.

The vast majority of the available experimental information is already explained by the SM. All accelerator physics is in agreement with the SM, except for neutrino oscillations. Presently only this neutrino physics, together with astrophysics and cosmology, gives us any phenomenological evidence for going beyond the SM.

One of the main goals of physics today is to find the fundamental theory beyond the SM. In first approximation we might ignore the indications of new physics and consider the possibility that the SM essentially represents physics well up to the Planck scale. Developing the ideas of Ref. [1], the authors of Ref. [2] suggested a scenario, using only the pure SM, in which an exponentially huge ratio between the fundamental (Planck) and electroweak scales results:

$$\frac{\mu_{fund}}{\mu_{ew}} \sim e^{40}. \tag{2}$$

This exponentially huge scale ratio occurs due to the required degeneracy of the three vacuum states (phases) discussed in Refs. [2–5].

In such a scenario it is reasonable to assume the existence of a simple and elegant postulate which helps us to explain the SM parameters: couplings, masses and mixing angles. In model [1, 2] such a postulate is based on a phenomenologically required result in cosmology [6]: the cosmological constant is zero, or approximately zero, meaning that the vacuum energy density is very small. *A priori* it is quite possible for a quantum field theory to have several minima of its effective potential as a function of its scalar fields. Postulating zero cosmological constant, we are confronted with a question: is the energy density, or cosmological constant, equal to zero (or approximately zero) for all possible vacua or it is zero only for that vacuum in which we live ?

This assumption would not be more complicated if we postulate that all the vacua which might exist in Nature, as minima of the effective potential, should have zero, or approximately zero cosmological constant. This postulate corresponds to what we call the Multiple Point Principle (MPP) [7–9].

MPP postulates: *there are many vacua with the same energy density or cosmological constant, and all cosmological constants are zero or approximately zero.*

There are circa 20 parameters in the SM characterizing the couplings and masses of the fundamental particles, whose values can only be understood in speculative models extending the SM. It was shown in Ref. [10] that the Family Replicated Gauge Group Model (FRGGM), suggested in Refs. [11,12] as an extension of the SM (see also reviews [13,14]), fits the SM fermion masses and mixing angles and describes all neutrino experimental data order of magnitude-wise using only 5 free parameters — five vacuum expectation values of the Higgs fields which break the FRGG symmetry to the SM. This approach based on the FRGG–model was previously called Anti–Grand Unified Theory (AGUT) and developed as a realistic alternative to SUSY Grand Unified Theories (GUTs) [11–21]. In Refs. [22–30] the MPP was applied to the investigation of phase transitions in the regularized gauge theories. A tiny order of magnitude of the cosmological constant was explained in a model involving supersymmetry breaking in N=1 supergravity and MPP [31,32]. The investigation of hierarchy problem in the SM extended by MPP and two Higgs doublets was developed in Ref. [33] (see also [34]). In the recent investigation [35] the MPP was applied to the flipped $SU(5) \times U(1)$ gauge theory.

The present paper is a review of the MPP implementation to phase transitions in different gauge theories.

2. The renormalization group equation for the effective potential

In the theory of a single scalar field interacting with a gauge field, the effective potential $V_{eff}(\phi_c)$ is a function of the classical field ϕ_c given by

$$V_{eff} = -\sum_{0}^{\infty} \frac{1}{n!}\Gamma^{(n)}(0)\phi_c^n, \tag{3}$$

where $\Gamma^{(n)}(0)$ is the one–particle irreducible (1PI) n–point Green's function calculated at zero external momenta. The renormalization group equation (RGE) for the effective potential means that the potential cannot depend on a change in the arbitrary renormalization scale parameter M:

$$\frac{dV_{eff}}{dM} = 0. \tag{4}$$

The effects of changing it are absorbed into changes in the coupling constants, masses and fields, giving so–called running quantities.

Considering the renormalization group (RG) improvement of the effective potential [36, 37] and choosing the evolution variable as

$$t = \log\left(\frac{\mu}{M}\right) = \log\left(\frac{\phi}{M}\right), \tag{5}$$

where μ is the energy scale, we have the Callan–Symanzik [38,39] RGE for the full $V_{eff}(\phi_c)$ with $\phi \equiv \phi_c$:

$$\left(M\frac{\partial}{\partial M} + \beta_{m^2}\frac{\partial}{\partial m^2} + \beta_\lambda \frac{\partial}{\partial \lambda} + \beta_g \frac{\partial}{\partial g} + \gamma\phi\frac{\partial}{\partial \phi}\right) V_{eff}(\phi) = 0, \tag{6}$$

where M is a renormalization mass scale parameter, β_{m^2}, β_λ, β_g are beta functions for the scalar mass squared m^2, scalar field self–interaction λ and gauge couplings g, respectively. Also γ is the anomalous dimension, and the gauge coupling constants: $g_i = (g',\, g,\, g_3)$ correspond to the $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ groups of the SM. Here the couplings depend on the renormalization scale M: $\lambda = \lambda(M)$, $m^2 = m^2(M)$ and $g_i = g_i(M)$. In general, we also consider the top quark Yukawa coupling $h \overset{def}{=} g_t$ and neglect the Yukawa couplings of all lighter fermions.

It is convenient to introduce a more compact notation for the parameters of theory. Define:

$$\lambda_p = \left(m^2,\, \lambda,\, g\right), \tag{7}$$

so that the RGE can be abbreviated as

$$\left(M\frac{\partial}{\partial M} + \beta_p\frac{\partial}{\partial \lambda_p} + \gamma\phi\frac{\partial}{\partial \phi}\right) V_{eff} = 0. \tag{8}$$

The general solution of the above-mentioned RGE has the following form [36]:

$$V_{eff} = -\frac{m^2(\phi)}{2}\left[G(t)\phi\right]^2 + \frac{\lambda(\phi)}{8}\left[G(t)\phi\right]^4 + C, \tag{9}$$

where

$$G(t) = \exp\left(-\int_0^t \gamma\left(t'\right) dt'\right). \tag{10}$$

We shall also use the notation $\lambda(t) = \lambda(\phi)$, $m^2(t) = m^2(\phi)$, $g_i^2(t) = g_i^2(\phi)$, which should not lead to any misunderstanding.

2.1. The second minimum of the Standard Model (SM) effective potential

In this Section our goal is to show the possible existence of a second (non–standard) minimum of the effective potential in the pure SM at the fundamental scale [1, 2]:

$$\phi_{min2} \gg v = \phi_{min1}. \tag{11}$$

The tree–level Higgs potential with the standard "weak scale minimum" at $\phi_{min1} = v$ is given by:

$$V\,(\text{tree–level}) = \frac{\lambda}{8}\left(\phi^2 - v^2\right)^2 + C. \tag{12}$$

In accord with cosmological results, we take the cosmological constants C for both vacua equal to zero (or approximately zero): $C = 0$ (or $C \approx 0$). The following requirements must be satisfied in order that the SM effective potential should have two degenerate minima:

$$V_{eff}\left(\phi^2_{min1}\right) = V_{eff}\left(\phi^2_{min2}\right) = 0, \tag{13}$$

$$V'_{eff}\left(\phi^2_{min1}\right) = V'_{eff}\left(\phi^2_{min2}\right) = 0, \tag{14}$$

where

$$V'(\phi^2) = \frac{\partial V}{\partial \phi^2}. \tag{15}$$

These degeneracy conditions first considered in Ref. [1] correspond to the MPP expectation. *The first minimum is the standard "Weak scale minimum", and the second one is the non–standard "Fundamental scale minimum" (if it exists).* An illustrative schematic picture of V_{eff} is presented in Fig. 1.

Here we consider the SM theory with zero temperature ($T = 0$). As was shown in Ref. [1], the above MPP–requirements lead to the condition that our electroweak vacuum is barely stable at $T = 0$.

With good accuracy, the predictions of Ref. [1] for the top quark and Higgs masses from the MPP requirement of a second degenerate vacuum, together with the identification of its position with the Planck scale $\phi_{min2} = M_{\rm Planck}$, were as follows:

$$M_t = 173 \pm 5\,\text{GeV}, \quad M_H = 135 \pm 9\,\text{GeV}. \tag{16}$$

Later, in Ref. [40], an alternative metastability requirement for the electroweak (first) vacuum was considered, which gave a Higgs mass prediction of 122 ± 11 GeV, close to the LEP lower bound of 115 GeV (Particle Data Group [41]).

Following Ref. [1], let us now investigate the conditions, Eqs. (13, 14), for the existence of a second degenerate vacuum at the fundamental scale: $\phi_{min2} \sim \mu_{fund.}$.

For large values of the Higgs field: $\phi^2 \gg m^2$ the effective potential V_{eff} is very well approximated by the quartic term in Eq. (9) and the degeneracy condition (13) gives:

$$\lambda\left(\phi_{min2}\right) = 0. \tag{17}$$

The condition (14) for a turning value then gives:

$$\lambda'\left(\phi_{min2}\right) = 0 \tag{18}$$

which can be expressed in the form:

$$\beta_\lambda\left(\phi_{min2},\, \lambda = 0\right) = 0. \tag{19}$$

In Ref. [2] the scale ϕ_{min2} depending on the experimental data uncertainties was calculated when the degeneracy conditions (17–19) were taken into account. For central values of experimentally observable quantities the result (2) was obtained and gave an exponentially huge ratio between the fundamental and electroweak scales.

3. Phase transitions. Triple point of water analogy

In general, it is quite possible that there exist a lot of vacua in Nature. If several vacua are degenerate then the phase diagram of theory contains *a special point* in which the corresponding phases meet together (see Fig. 2). This special point is the Multiple Critical Point (MCP). The phase diagram of any gauge theory is presented by a space which has axes given by bare coupling constants (and maybe by bare masses).

Here it is useful to remind you a triple point of water analogy.

It is well known in the thermal physics that in the range of fixed extensive quantities: volume, energy and a number of moles the degenerate phases of water (namely, ice, water and vapour presented in Fig. 3) exist on the phase diagram (P, T) of Fig. 4 at the fine-tuned values of the intensive variables — pressure P and temperature T:

$$T_c \approx 0.01\,^{\circ}\mathrm{C} \quad \text{and} \quad P_c \approx 4.58\,\mathrm{mm\ Hg}, \tag{20}$$

giving the critical (triple) point O shown in Fig. 4. This is a triple point of water analogy.

The idea of the Multiple Point Principle has its origin from the lattice investigations of gauge theories. In particular, Monte Carlo simulations of $U(1)$–, $SU(2)$– and $SU(3)$–gauge theories on lattice indicate the existence of the triple critical point.

4. Lattice theories

4.1. Mathematical structure of lattice gauge theories

A lattice contains sites, links and plaquettes. Link variables defined on the edges of the lattice are fundamental variables of the lattice theory. These variables are simultaneously the elements of the gauge group G, describing a symmetry of the corresponding lattice gauge theory:

$$\mathcal{U}(x \bullet\!\!\longrightarrow\!\!\bullet\, y) \in G. \tag{21}$$

It is easy to understand the sense of this variable turning to the differential geometry of the continuum space–time in which our gauge fields exist. Such a space geometrically is equivalent to curvilinear space and an operator, which compares fields at different points, is an operator of the parallel transport between the points x, y:

$$\mathcal{U}(x,y) = Pe^{ig\int_{C_{xy}} A_\mu(x)dx^\mu}, \tag{22}$$

where P is the path ordering operator and C_{xy} is a curve from point x till point y. Moreover, the operator:

$$W = \mathrm{Tr}\left(Pe^{ig\oint_C A_\mu(x)dx^\mu}\right) \tag{23}$$

is the well–known Wilson–loop. In the case of scalar field $\phi(x)$, interacting with gauge field A_μ, we have an additional gauge invariant observable:

$$\phi^{+}(y)\left[Pe^{ig\int_{C_{xy}}A_\mu(x)dx^\mu}\right]\phi(x). \tag{24}$$

The link variable (21) is a lattice version of Eq. (22):

$$\mathcal{U}(x \bullet\!\!-\!\!\!-\!\!\bullet\, y) = e^{i\Theta_\mu(n)} \equiv \mathcal{U}_\mu(n). \tag{25}$$

This link variable connects the point n and the point $n{+}a_\mu$, where the index μ indicates the direction of a link in the hypercubic lattice with parameter a. Considering the infinitesimal increment of the operator (25) in the continuum limit, we have:

$$\Theta_\mu(n) = a\hat{A}_\mu(x), \tag{26}$$

where the quantity

$$\hat{A}_\mu(x) = gA^j_\mu(x)t^j \tag{27}$$

contains the generator t^j of the group G if $G = SU(N)$.

For $G = SU(3)$ we have $t^j = \lambda^j/2$, where λ^j are the well–known Gell–Mann matrices.

For $G = U(1)$:

$$\hat{A}_\mu(x) = gA_\mu(x). \tag{28}$$

Plaquette variables are not independent because they are products of link variables:

$$\mathcal{U}_p \equiv \mathcal{U}(\square) \stackrel{def}{=} \mathcal{U}\left(\square\right)\mathcal{U}\left(\square\right)\mathcal{U}\left(\square\right)\mathcal{U}\left(\square\right). \tag{29}$$

4.2. Lattice actions

The lattice action $S[\mathcal{U}]$ is invariant under the gauge transformations on a lattice

$$\mathcal{U}(x \bullet\!\!-\!\!\!-\!\!\bullet\, y) \longrightarrow \Lambda(x)\mathcal{U}(x \bullet\!\!-\!\!\!-\!\!\bullet\, y)\Lambda^{-1}(y), \tag{30}$$

where $\Lambda(x) \in G$.

The simplest action $S[\mathcal{U}]$ is given by the expression:

$$S[\mathcal{U}] = \sum_q \frac{\beta_q}{\dim q}\sum_p Re\left(\mathrm{Tr}\left(\mathcal{U}_p^{(q)}\right)\right). \tag{31}$$

Here q is the index of the representation of the group G, $\dim q$ is the dimension of this representation, and $\beta_q = 1/g_q^2$, where g_q is the coupling constant of gauge fields corresponding to the representation q.

The path integral

$$Z = \int D\mathcal{U}(\bullet\!\!-\!\!\!-\!\!\bullet)e^{-S[\mathcal{U}(\bullet\!\!-\!\!\!-\!\!\bullet)]}, \tag{32}$$

which is an analogue of the partition function, describes the lattice gauge theory in the Euclidean four–dimensional space.

It is necessary to construct the lattice field theory such that, for $a \to 0$, i.e. in the continuum limit, it leads to a regularized smooth gauge theory of fields $A_\mu^j(x)$, where j is the symmetry subscript. In the opposite case, a passage to the continuum limit is not unique [42].

Let us consider the simplest case of the group $G = U(1)$, using the only representation of this group in Eq. (31):

$$S\left[\mathcal{U}_p\right] = \beta \sum_p Re\left(\mathcal{U}_p\right). \tag{33}$$

Here the quantity $\mathcal{U}_p$ is given by Eq. (29) in which the link variables $\mathcal{U}(\ \bullet\!\!-\!\!\!-\!\!\bullet\ )$ are complex numbers with their moduli equal to unity, i.e.,

$$\mathcal{U}(x\ \bullet\!\!-\!\!\!-\!\!\bullet\ y) = \left\{z | z \in \mathbf{C},\, |z| = 1\right\}. \tag{34}$$

In the lattice model, the Lorentz gauge condition has the form

$$\prod_{x\ \bullet\!-\!\!-\!\bullet\ y} \mathcal{U}(x\ \bullet\!\!-\!\!\!-\!\!\bullet\ y) = 1. \tag{35}$$

Introducing the notation

$$z = e^{i\Theta}, \tag{36}$$

we can write:

$$\mathcal{U}_p = e^{i\Theta_p}. \tag{37}$$

The variables $\mathcal{U}_p$ are not independent, they satisfy the identity:

$$\prod_{\Box \in (\text{lattice cube})} \mathcal{U}\left(\Box\right) = I, \tag{38}$$

called the Bianchi identity. In Eq. (38) the product is taken over all plaquettes belonging to the cell (cube) of the hypercubic lattice.

According to Eqs. (33) and (37), the simplest lattice $U(1)$ action has the form:

$$S\left[\mathcal{U}_p\right] = \beta \sum_p \cos\Theta_p. \tag{39}$$

For the compact lattice QED: $\beta = 1/e_0^2$, where e_0 is the bare electric charge.

The lattice $SU(N)$ gauge theories were first introduced by K. Wilson [43] for studying the problem of confinement. He suggested the following simplest action:

$$S = -\frac{\beta}{N} \sum_p Re\left(\mathrm{Tr}\left(\mathcal{U}_p\right)\right), \tag{40}$$

where the sum runs over all plaquettes of a hypercubic lattice and $\mathcal{U}_p \equiv U(\Box)$ belongs to the adjoint representation of $SU(N)$.

Monte Carlo simulations of these simple Wilson lattice theories in four dimensions showed a (or an almost) second–order deconfining phase transition for $U(1)$ [44, 45], a crossover behaviour for $SU(2)$ and $SU(3)$ [46, 47], and a first–order phase transition for $SU(N)$ with $N \geq 4$ [48].

Bhanot and Creutz [49,50] have generalized the simple Wilson theory, introducing two parameters in the $SU(N)$ action:

$$S = \sum_p \left[-\frac{\beta_f}{N} Re\left(\mathrm{Tr}\left(\mathcal{U}_p\right)\right) - \frac{\beta_A}{N^2-1} Re\left(\mathrm{Tr}_A\left(\mathcal{U}_p\right)\right) \right], \tag{41}$$

where β_f, Tr and β_A, Tr_A are respectively the lattice constants and traces in the fundamental and adjoint representations of $SU(N)$.

The phase diagrams, obtained for the generalized lattice $SU(2)$ and $SU(3)$ theories (41) by Monte Carlo methods in Refs. [49,50] (see also [51]) are shown in Figs. 5, 6. They indicate the existence of a triple point which is a boundary point of three first–order phase transitions: the "Coulomb–like" and $SU(N)/Z_N$ and Z_N confinement phases meet together at this point. From the triple point emanate three phase border lines which separate the corresponding phases. The Z_N phase transition is a discrete transition, occurring when lattice plaquettes jump from the identity to nearby elements in the group. The $SU(N)/Z_N$ phase transition is due to a condensation of monopoles (a consequence of the non–trivial Π_1 of the group).

The phase diagram of the lattice gauge theory described by the action with mixed $SU(2)$–$SO(3)$ symmetries [52] is presented in Fig. 7. Here I is a range where the densities of Z_2–vortices (E) and Z_2–monopoles (M) accept the values $E \sim M \sim 0.5$. The range II corresponds to $E \sim 0.5$, $M \sim 0$ and in the range III we have $E \sim M \sim 0$. The closed Z_2–vortex and Z_2–monopole (in the three–dimensional lattice) are shown in Fig. 8 and Fig. 9, respectively.

Monte Carlo simulations of the $U(1)$ gauge theory, described by the two-parameter lattice action [53, 54]:

$$S = \sum_p \left[\beta^{lat} \cos\Theta_p + \gamma^{lat}\cos 2\Theta_p\right], \quad \text{where} \quad \mathcal{U}_p = e^{i\Theta_p}, \tag{42}$$

also indicate the existence of a triple point on the corresponding phase diagram: "Coulomb–like", totally confining and Z_2 confining phases come together at this triple point (see Fig. 10).

In general, we have a number of phases meeting at the MCP. For example, Fig. 11 demonstrates the meeting of the five phases in the case of the gauge theory with the $U(1) \times SU(2)$ symmetry considered in Ref. [8].

Recently N. Arkani–Hamed [55] referred to the modern cosmological theory which assumes the existence of a lot of degenerate vacua in the Universe.

In Ref. [56] the coexistence of different quantum vacua of our Universe is explained by the MPP. It was shown that these vacua are regulated by the baryonic charge and all the coexisting vacua exhibit the baryonic asymmetry. The present baryonic asymmetry of the Universe is discussed.

Lattice theories are given in reviews [42]. The next efforts of the lattice simulations of the $SU(N)$ gauge theories are presented in the review [57], etc.

4.3. Lattice artifact monopoles

Lattice monopoles are responsible for the confinement in lattice gauge theories what is confirmed by many numerical and theoretical investigations (see reviews [58] and papers [59]).

In the compact lattice gauge theory the monopoles are not physical objects: they are lattice artifacts driven to infinite mass in the continuum limit. Weak coupling ("Coulomb") phase terminates because of the appearance for non–trivial topological configurations which are able to change the vacuum. These topological excitations are closed monopole loops (or universe lines of monopole–anti–monopole pairs). When these monopole loops are long and numerous, they are responsible for the confinement. But when they are dilute and small, the Coulomb ("free photon") phase appears. Banks et al. [60] have shown that in the Villain form of the $U(1)$ lattice gauge theory [61] it is easy to exhibit explicitly the contribution of the topological excitations. The Villain lattice action is:

$$S_V = \frac{\beta}{2}\sum_p \left(\theta_p - 2\pi k\right)^2, \quad k \in Z. \tag{43}$$

In such a model the partition function Z may be written in a factorized form:

$$Z = Z_C Z_M, \tag{44}$$

where Z_C is a part describing the photons:

$$Z_C \sim \int\limits_{-\infty}^{\infty} d\Theta \exp\left[-\frac{\beta}{2}\sum_{\Box}\Theta^2\left(\Box\right)\right], \tag{45}$$

and Z_M is the partition function of a gas of the monopoles [62, 63]:

$$Z_M \sim \sum_{m\in Z}\exp\left[-2\pi^2\beta\sum_{x,y} m(x)v(x-y)m(y)\right]. \tag{46}$$

In Eq. (46) $v(x-y)$ is a lattice version of $1/r$–potential and $m(x)$ is the charge of the monopole, sitting in an elementary cube c of the dual lattice, which can be simply expressed in terms of the integer variables n_p :

$$m = \sum_{p\in\partial c} n_p, \tag{47}$$

where n_p is a number of Dirac strings passing through the plaquettes of the cube c.

The Gaussian part Z_C provides the usual Coulomb potential, while the monopole part Z_M leads, at large separations, to a linearly confining potential.

It is more complicated to exhibit the contribution of monopoles even in the $U(1)$ lattice gauge theory described by the simple Wilson action (39). Let us consider the Wilson loop as a rectangle of length T in the 1–direction (time) and width R in the 2–direction (space–like distance), then we can extract the potential $V(R)$ between two static charges of opposite signs:

$$V(R) = -\lim_{T\to\infty}\frac{1}{T}\log(\langle W\rangle), \tag{48}$$

and obtain:

$$V(R) = -\frac{\alpha(\beta)}{R} \quad - \quad \text{in “Coulomb” phase,} \tag{49}$$

$$V(R) = \sigma R - \frac{\alpha(\beta)}{R} + O\left(\frac{1}{R^3}\right) + \text{const} - \text{in confinement phase.} \tag{50}$$

4.4. The behaviour of electric fine structure constant α near the phase transition point. “Freezing” of α

The lattice investigators were not able to obtain the lattice triple point values of $\alpha_{i,\,crit}$ for $i = 2,\, 3$ by Monte Carlo simulations method. Only the critical value of the electric fine structure constant $\alpha(\beta)$ was obtained in Ref. [54] in the compact QED described by the Wilson and Villain actions (39) and (43) respectively:

$$\alpha_{crit}^{lat} = 0.20 \pm 0.015 \quad \text{and} \quad \tilde{\alpha}_{crit}^{lat} = 1.25 \pm 0.10 \quad \text{at} \quad \beta_T \equiv \beta_{crit} \approx 1.011. \tag{51}$$

Here

$$\alpha = \frac{e^2}{4\pi} \quad \text{and} \quad \tilde{\alpha} = \frac{g^2}{4\pi}, \tag{52}$$

where g is a magnetic charge of monopoles.

Using the Dirac relation for elementary charges (see below Subsection 11.1), we have:

$$eg = 2\pi, \quad \text{or} \quad \alpha\tilde{\alpha} = \frac{1}{4}. \tag{53}$$

The behaviour of $\alpha(\beta)$ in the vicinity of the phase transition point β_T (given by Ref. [54]) is shown in Fig. 12 for the Wilson and Villain lattice actions. Fig. 13 demonstrates the

comparison of the function $\alpha(\beta)$ obtained by Monte Carlo method for the Wilson lattice action and by theoretical calculation of the same quantity. The theoretical (dashed) curve was calculated by so–called "Parisi improvement formula" [64]:

$$\alpha(\beta) = [4\pi\beta W_p]^{-1}. \tag{54}$$

Here $W_p = \langle \cos\Theta_p \rangle$ is a mean value of the plaquette energy. The corresponding values of W_p are taken from Ref. [53].

The theoretical value of α_{crit} is less than the "experimental" (Monte Carlo) value (51):

$$\alpha_{crit}\,(\text{lattice theory}) \approx 0.12. \tag{55}$$

This discrepancy between the theoretical and "experimental" results is described by monopole contributions: the fine structure constant α is renormalised by an amount proportional to the susceptibility of the monopole gas [62]:

$$K = \frac{\alpha_{crit}\,(\text{Monte Carlo})}{\alpha_{crit}\,(\text{lattice theory})} \approx \frac{0.20}{0.12} \approx 1.66. \tag{56}$$

Such an enhancement of the critical fine structure constant is due to vacuum monopole loops [63].

According to Fig. 14:

$$\alpha^{-1}_{crit.,\,theor.} \approx 8.5. \tag{57}$$

This result does not coincide with the lattice result (51) which gives the following value:

$$\alpha^{-1}_{crit.,\,theor.} \approx 5. \tag{58}$$

The deviation of theoretical calculations of $\alpha(\beta)$ from the lattice ones, which is shown in Figs. 13, 14, has the following explanation: "Parisi improvement formula" (54) is valid in Coulomb phase where the mass of artifact monopoles is infinitely large and photon is massless. But in the vicinity of the phase transition (critical) point the monopole mass $m \to 0$ and photon acquires the non–zero mass $m_0 \neq 0$ in the confinement range. This phenomenon leads to the "freezing" of α: the effective electric fine structure constant is almost unchanged in the confinement phase and approaches to its maximal value $\alpha = \alpha_{max}$. The authors of Ref. [65] predicted that in the confinement phase, where we have the formation of strings, the fine structure constant α cannot be infinitely large, but has the maximal value:

$$\alpha_{max} = \frac{\pi}{12} \approx 0.26, \tag{59}$$

due to the Casimir effect for strings. The authors of Ref. [66] developed this viewpoint in the spinor QED: the vacuum polarization induced by thin "strings"–vortices of the magnetic flux leads to the suggestion of an analogue of the "spaghetti vacuum" [67] as a possible mechanism for avoiding the divergences in the perturbative QED. According

to Ref. [66], the non–perturbative sector of QED arrests the growth of the effective α to infinity and confirms the existence of α_{max}.

We see that Fig. 12 demonstrates the tendency to freezing of α in the compact QED.

The analogous "freezing" of α_s was considered in QCD in Ref. [68].

5. The Higgs Monopole Model and phase transition in the regularized $U(1)$ gauge theory

The simplest effective dynamics describing the confinement mechanism in the pure gauge lattice $U(1)$ theory is the dual Abelian Higgs model of scalar monopoles [69] (see also Refs. [58] and [59]).

In the previous papers [8] and [20] the calculations of the $U(1)$ phase transition (critical) coupling constant were connected with the existence of artifact monopoles in the lattice gauge theory and also in the Wilson loop action model [20].

In Ref. [20] the authors have put forward the speculations of Refs. [8] and [13] suggesting that the modifications of the form of the lattice action might not change too much the phase transition value of the effective continuum coupling constant. The purpose was to investigate this approximate stability of the critical coupling with respect to a somewhat new regularization being used instead of the lattice, rather than just modifying the lattice in various ways. In [20] the Wilson loop action was considered in the approximation of circular loops of radii $R \geq a$. It was shown that the phase transition coupling constant is indeed approximately independent of the regularization method: $\alpha_{crit} \approx 0.204$, in correspondence with the Monte Carlo simulation result on lattice: $\alpha_{crit} \approx 0.20 \pm 0.015$ (see Eq. (51)).

But in Refs. [22–29] instead of using the lattice or Wilson loop cut–off we have considered the Higgs Monopole Model (HMM) approximating the lattice artifact monopoles as fundamental point–like particles described by the Higgs scalar fields. Considering the renormalization group improvement of the effective Coleman–Weinberg potential [36,37], written in Ref. [23] for the dual sector of scalar electrodynamics in the two–loop approximation, we have calculated the $U(1)$ critical values of the magnetic fine structure constant:

$$\tilde{\alpha}_{crit} = \frac{g_{crit}^2}{4\pi} \approx 1.20 \tag{60}$$

and electric fine structure constant

$$\alpha_{crit} = \frac{\pi}{g_{crit}^2} \approx 0.208 \quad \text{(by the Dirac relation)}\,. \tag{61}$$

These values coincide with the lattice result (51). The next Subsections follow the review of the HMM calculations of the $U(1)$ critical couplings obtained in Refs. [23–25].

5.1. The Coleman–Weinberg effective potential for the HMM

As it was mentioned above, the dual Abelian Higgs model of scalar monopoles (shortly HMM) describes the dynamics of the confinement in lattice theories. This model, first suggested in Ref. [69], considers the following Lagrangian:

$$L = -\frac{1}{4g^2}F^2_{\mu\nu}(B) + \frac{1}{2}\left|(\partial_\mu - iB_\mu)\,\Phi\right|^2 - U(\Phi),$$

where

$$U(\Phi) = \frac{1}{2}\mu^2|\Phi|^2 + \frac{\lambda}{4}|\Phi|^4 \tag{62}$$

is the Higgs potential of scalar monopoles with magnetic charge g, and B_μ is the dual gauge (photon) field interacting with the scalar monopole field Φ. In this theory the parameter μ^2 is negative. In Eq. (62) the complex scalar field Φ contains the Higgs (ϕ) and Goldstone (χ) boson fields:

$$\Phi = \phi + i\chi. \tag{63}$$

The effective potential in the Higgs model of scalar electrodynamics was first calculated by Coleman and Weinberg [36] in the one–loop approximation. The general method of its calculation is given in the review [37]. Using this method, we can construct the effective potential for HMM. In this case the total field system of the gauge (B_μ) and magnetically charged (Φ) fields is described by the partition function which has the following form in Euclidean space:

$$Z = \int [DB][D\Phi]\left[D\Phi^+\right] e^{-S}, \tag{64}$$

where the action $S = \int d^4x L(x) + S_{gf}$ contains the Lagrangian (62) written in Euclidean space and gauge fixing action S_{gf}.

Let us consider now a shift:

$$\Phi(x) = \Phi_b + \hat{\Phi}(x) \tag{65}$$

with Φ_b as a background field and calculate the following expression for the partition function in the one-loop approximation:

$$\begin{aligned} Z &= \int [DB]\left[D\hat{\Phi}\right]\left[D\hat{\Phi}^+\right] \exp\left\{-S\left(B, \Phi_b\right) - \int d^4x \left[\left.\frac{\delta S(\Phi)}{\delta\Phi(x)}\right|_{\Phi=\Phi_b} \hat{\Phi}(x) + h.c.\right]\right\} \\ &= \exp\left\{-F\left(\Phi_b,\, g^2,\, \mu^2,\, \lambda\right)\right\}. \end{aligned} \tag{66}$$

Using the representation (63), we obtain the effective potential:

$$V_{eff} = F\left(\phi_b,\, g^2,\, \mu^2,\, \lambda\right) \tag{67}$$

given by the function F of Eq. (66) for the real constant background field $\Phi_b = \phi_b = \text{const}$. In this case the one–loop effective potential for monopoles coincides with the expression

of the effective potential calculated by the authors of Ref. [36] for scalar electrodynamics and extended to the massive theory (see review [37]):

$$\begin{aligned} V_{eff}(\phi_b^2) &= \frac{\mu^2}{2}{\phi_b}^2 + \frac{\lambda}{4}{\phi_b}^4 + \frac{1}{64\pi^2}\left[3g^4{\phi_b}^4\log\left(\frac{\phi_b^2}{M^2}\right) + \left(\mu^2 + 3\lambda{\phi_b}^2\right)^2\log\left(\frac{\mu^2+3\lambda\phi_b^2}{M^2}\right)\right. \\ &\left. + \left(\mu^2+\lambda\phi_b^2\right)^2\log\left(\frac{\mu^2+\lambda\phi_b^2}{M^2}\right)\right] + C, \end{aligned} \tag{68}$$

where M is the cut–off scale and C is a constant not depending on ϕ_b^2.

The effective potential (67) has several minima. Their position depends on g^2, μ^2 and λ. If the first local minimum occurs at $\phi_b = 0$ and $V_{eff}(0) = 0$, it corresponds to the so–called "symmetrical phase", which is the Coulomb–like phase in our description. Then it is easy to determine the constant C in Eq. (68):

$$C = -\frac{\mu^4}{16\pi^2}\log\left(\frac{\mu}{M}\right), \tag{69}$$

and we have the effective potential for HMM described by the following expression:

$$V_{eff}(\phi_b^2) = \frac{\mu_{run}^2}{2}\phi_b^2 + \frac{\lambda_{run}}{4}\phi_b^4 + \frac{\mu^4}{64\pi^2}\log\left(\frac{(\mu^2+3\lambda\phi_b^2)\,(\mu^2+\lambda\phi_b^2)}{\mu^4}\right). \tag{70}$$

Here λ_{run} is the running scalar field self–interaction constant given by the expression standing in front of ϕ_b^4 in Eq. (68):

$$\lambda_{run}(\phi_b^2) = \lambda + \frac{1}{16\pi^2}\left[3g^4\log\left(\frac{\phi_b^2}{M^2}\right) + 9\lambda^2\log\left(\frac{\mu^2+3\lambda\phi_b^2}{M^2}\right) + \lambda^2\log\left(\frac{\mu^2+\lambda\phi_b^2}{M^2}\right)\right]. \tag{71}$$

The running squared mass of the Higgs scalar monopoles also follows from Eq. (68):

$$\mu_{run}^2(\phi_b^2) = \mu^2 + \frac{\lambda\mu^2}{16\pi^2}\left[3\log\left(\frac{\mu^2+3\lambda\phi_b^2}{M^2}\right) + \log\left(\frac{\mu^2+\lambda\phi_b^2}{M^2}\right)\right]. \tag{72}$$

As it was shown in Ref. [36], the effective potential can be improved by consideration of the renormalization group equation (RGE).

5.2. Renormalization group equations in the HMM

The RGE for the effective potential are given by Eqs. (6–10). A set of ordinary differential equations (RGE) corresponds to Eq. (6):

$$\frac{d\lambda_{run}}{dt} = \beta_\lambda\left(g_{run}(t),\,\lambda_{run}(t)\right), \tag{73}$$

$$\frac{d\mu_{run}^2}{dt} = \mu_{run}^2(t)\beta_{(\mu^2)}\left(g_{run}(t),\,\lambda_{run}(t)\right), \tag{74}$$

$$\frac{dg_{run}^2}{dt} = \beta_g\left(g_{run}(t),\,\lambda_{run}(t)\right). \tag{75}$$

So far as the mathematical structure of HMM is equivalent to the Higgs scalar electrodynamics, we can use all results of the last theory in our calculations, replacing the electric charge e and photon field A_μ by magnetic charge g and dual gauge field B_μ.

Let us write now the one–loop potential (70) as

$$V_{eff} = V_0 + V_1, \tag{76}$$

where

$$V_0 = \frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4,$$

$$\begin{aligned} V_1 &= \frac{1}{64\pi^2}\left[3g^4\phi^4 \log\left(\frac{\phi^2}{M^2}\right) + \left(\mu^2 + 3\lambda\phi^2\right)^2 \log\left(\frac{\mu^2 + 3\lambda\phi^2}{M^2}\right)\right. \\ &\quad \left. + \left(\mu^2 + \lambda\phi^2\right)^2 \log\left(\frac{\mu^2 + \lambda\phi^2}{M^2}\right) - 2\mu^4 \log\left(\frac{\mu^2}{M^2}\right)\right]. \end{aligned} \tag{77}$$

We can plug this V_{eff} into RGE (6) and obtain the following equation (see [37]):

$$\left(\beta_\lambda \frac{\partial}{\partial\lambda} + \beta_{(\mu^2)}\mu^2 \frac{\partial}{\partial\mu^2} - \gamma\phi^2 \frac{\partial}{\partial\phi^2}\right) V_0 = -M^2 \frac{\partial V_1}{\partial M^2}. \tag{78}$$

Equating ϕ^2 and ϕ^4 coefficients, we obtain the expressions of β_λ and $\beta_{(\mu^2)}$ in the one–loop approximation:

$$\beta_\lambda^{(1)} = 2\gamma\lambda_{run} + \frac{5\lambda_{run}^2}{8\pi^2} + \frac{3g_{run}^4}{16\pi^2}, \tag{79}$$

$$\beta_{(\mu^2)}^{(1)} = \gamma + \frac{\lambda_{run}}{4\pi^2}. \tag{80}$$

The one–loop result for γ is given in Ref. [36] for scalar field with electric charge e, but it is easy to rewrite this γ–expression for monopoles with charge $g = g_{run}$:

$$\gamma^{(1)} = -\frac{3g_{run}^2}{16\pi^2}. \tag{81}$$

Finally we have:

$$\frac{d\lambda_{run}}{dt} \approx \beta_\lambda^{(1)} = \frac{1}{16\pi^2}\left(3g_{run}^4 + 10\lambda_{run}^2 - 6\lambda_{run}g_{run}^2\right), \tag{82}$$

$$\frac{d\mu_{run}^2}{dt} \approx \beta_{(\mu^2)}^{(1)} = \frac{\mu_{run}^2}{16\pi^2}\left(4\lambda_{run} - 3g_{run}^2\right). \tag{83}$$

The expression of β_g–function in the one–loop approximation also is given by the results of Ref. [36]:

$$\frac{dg_{run}^2}{dt} \approx \beta_g^{(1)} = \frac{g_{run}^4}{48\pi^2}. \tag{84}$$

The RG β–functions for different renormalizable gauge theories with semisimple group have been calculated in the two–loop approximation [70–75] and even beyond [76]. But in this paper we made use the results of Refs. [70] and [73] for calculation of β–functions and

anomalous dimension in the two–loop approximation, applied to the HMM with scalar monopole fields. The higher approximations essentially depend on the renormalization scheme [76]. Thus, on the level of two–loop approximation we have for all β–functions:

$$\beta = \beta^{(1)} + \beta^{(2)}, \tag{85}$$

where

$$\beta_\lambda^{(2)} = \frac{1}{(16\pi^2)^2}\left(-25\lambda^3 + \frac{15}{2}g^2\lambda^2 - \frac{229}{12}g^4\lambda - \frac{59}{6}g^6\right). \tag{86}$$

and

$$\beta_{(\mu^2)}^{(2)} = \frac{1}{(16\pi^2)^2}\left(\frac{31}{12}g^4 + 3\lambda^2\right). \tag{87}$$

The gauge coupling $\beta_g^{(2)}$–function is given by Ref. [70]:

$$\beta_g^{(2)} = \frac{g^6}{(16\pi^2)^2}. \tag{88}$$

Anomalous dimension follows from calculations made in Ref. [73]:

$$\gamma^{(2)} = \frac{1}{(16\pi^2)^2}\frac{31}{12}g^4. \tag{89}$$

In Eqs. (85–89) and below, for simplicity, we have used the following notations: $\lambda \equiv \lambda_{run}$, $g \equiv g_{run}$ and $\mu \equiv \mu_{run}$.

5.3. The phase diagram in the HMM

Let us apply the effective potential calculation as a technique for the getting phase diagram information for the condensation of monopoles in HMM. As it was mentioned in the Subsection 5.1., the effective potential (67) can have several minima. Their positions depend on g^2, μ^2 and λ:

$$\phi_0 = \phi_{min1} = f\left(g^2,\, \mu,\, \lambda\right). \tag{90}$$

The first local minimum at $\phi_0 = 0$ and $V_{eff}(0) = 0$ corresponds to the "symmetric", or Coulomb–like phase, presented in Fig. 15. In the case when the effective potential has the second local minimum at $\phi_0 = \phi_{min2} \neq 0$ with $V_{eff}^{min}\left(\phi_{min2}^2\right) < 0$, we have the confinement phase (see Fig. 16). The phase transition between the Coulomb–like and confinement phases is given by degeneracy of the first local minimum (at $\phi_0 = 0$) with the second minimum (at $\phi_0 = \phi_{min2}$). These degenerate minima are shown in Fig. 17 by the solid curve 1. They correspond to the different vacua arising in the present model. The dashed curve 2 in Fig. 17 describes the appearance of two minima corresponding to the confinement phases (see details in Subsection 5.5.).

The conditions of the existence of degenerate vacua are given by the following equations:

$$V_{eff}(0) = V_{eff}\left(\phi_0^2\right) = 0, \tag{91}$$

$$\left.\frac{\partial V_{eff}}{\partial \phi}\right|_{\phi=0} = \left.\frac{\partial V_{eff}}{\partial \phi}\right|_{\phi=\phi_0} = 0, \quad \text{or} \quad V'_{eff}\left(\phi_0^2\right) \equiv \left.\frac{\partial V_{eff}}{\partial \phi^2}\right|_{\phi=\phi_0} = 0, \tag{92}$$

and inequalities

$$\left.\frac{\partial^2 V_{eff}}{\partial \phi^2}\right|_{\phi=0} > 0, \qquad \left.\frac{\partial^2 V_{eff}}{\partial \phi^2}\right|_{\phi=\phi_0} > 0. \tag{93}$$

The first equation (91), applied to Eq. (9), gives:

$$\mu_{run}^2 = -\frac{1}{2}\lambda_{run}\left(t_0\right)\phi_0^2 G^2\left(t_0\right), \quad \text{where} \quad t_0 = \log\left(\frac{\phi_0^2}{M^2}\right). \tag{94}$$

Calculating the first derivative of V_{eff} given by Eq. (92), we obtain the following expression:

$$\begin{aligned} V'_{eff}\left(\phi^2\right) &= \frac{V_{eff}\left(\phi^2\right)}{\phi^2}\left(1 + 2\frac{d\log(G)}{dt}\right) + \frac{1}{2}\frac{d\mu_{run}^2}{dt}G^2(t) \\ &+ \frac{1}{4}\left(\lambda_{run}(t) + \frac{d\lambda_{run}}{dt} + 2\lambda_{run}\frac{d\log(G)}{dt}\right)G^4(t)\phi^2. \end{aligned} \tag{95}$$

From Eq. (10) we have:

$$\frac{d\log(G)}{dt} = -\frac{1}{2}\gamma. \tag{96}$$

It is easy to find the joint solution of equations

$$V_{eff}\left(\phi_0^2\right) = V'_{eff}\left(\phi_0^2\right) = 0. \tag{97}$$

Using RGE (73), (74) and Eqs. (94–96), we obtain:

$$V'_{eff}\left(\phi_0^2\right) = \frac{1}{4}\left(-\lambda_{run}\beta_{(\mu^2)} + \lambda_{run} + \beta_\lambda - \gamma\lambda_{run}\right)G^4\left(t_0\right)\phi_0^2 = 0, \tag{98}$$

or

$$\beta_\lambda + \lambda_{run}\left(1 - \gamma - \beta_{(\mu^2)}\right) = 0. \tag{99}$$

Putting into Eq. (99) the functions $\beta_\lambda^{(1)}$, $\beta_{(\mu^2)}^{(1)}$ and $\gamma^{(1)}$ given by Eqs. (79–81) and (84), we obtain in the one–loop approximation the following equation for the phase transition border:

$$g_{PT}^4 = -2\lambda_{run}\left(\frac{8\pi^2}{3} + \lambda_{run}\right). \tag{100}$$

The curve (100) is represented on the phase diagram $(\lambda_{run};\, g_{run}^2)$ of Fig. 18 by the curve “1” which describes the border between the “Coulomb–like” phase with $V_{eff} \geq 0$ and the confinement one with $V_{eff}^{min} < 0$. This border corresponds to the one–loop approximation.

Using Eqs. (79–81) and (84–89), we are able to construct the phase transition border in the two–loop approximation. Substituting these equations into Eq. (99), we obtain the following phase transition border curve equation in the two–loop approximation:

$$3y^2 - 16\pi^2 + 6x^2 + \frac{1}{16\pi^2}\left(28x^3 + \frac{15}{2}x^2 y + \frac{97}{4}xy^2 - \frac{59}{6}y^3\right) = 0, \tag{101}$$

where $x = -\lambda_{PT}$ and $y = g^2_{PT}$ are the phase transition values of $-\lambda_{run}$ and g^2_{run}. Choosing the physical branch corresponding to $g^2 \geq 0$ and $g^2 \to 0$ when $\lambda \to 0$, we have received the curve 2 on the phase diagram (λ_{run}; g^2_{run}) shown in Fig. 18. This curve corresponds to the two–loop approximation and can be compared with the curve 1 of Fig. 18, which describes the same phase border calculated in the one–loop approximation. It is easy to see that the accuracy of the one–loop approximation is not excellent and can commit errors of order 30%.

According to the phase diagram drawn in Fig. 18, the confinement phase begins at $g^2 = g^2_{max}$ and exists under the phase transition border line in the region $g^2 \leq g^2_{max}$, where e^2 is large: $e^2 \geq (2\pi/g_{max})^2$ due to the Dirac relation (see Eq. (53)). Therefore, we have:

$$\begin{aligned} g^2_{crit} &= g^2_{max1} \approx 18.61 \quad - \quad \text{in the one–loop approximation,} \\ g^2_{crit} &= g^2_{max2} \approx 15.11 \quad - \quad \text{in the two–loop approximation.} \end{aligned} \tag{102}$$

We see the deviation of results of order 20%. The results (102) give:

$$\begin{aligned} \tilde{\alpha}_{crit} &= \frac{g^2_{crit}}{4\pi} \approx 1.48 \quad - \quad \text{in the one–loop approximation,} \\ \tilde{\alpha}_{crit} &= \frac{g^2_{crit}}{4\pi} \approx 1.20 \quad - \quad \text{in the two–loop approximation.} \end{aligned} \tag{103}$$

Using the Dirac relation (53): $\alpha\tilde{\alpha} = 1/4$, we obtain the following values for the critical electric fine structure constant:

$$\begin{aligned} \alpha_{crit} &= \frac{1}{4\tilde{\alpha}_{crit}} \approx 0.17 \quad - \quad \text{in the one–loop approximation,} \\ \alpha_{crit} &= \frac{1}{4\tilde{\alpha}_{crit}} \approx 0.208 \quad - \quad \text{in the two–loop approximation.} \end{aligned} \tag{104}$$

The last result coincides with the lattice values (51) obtained for the compact QED by Monte Carlo method [54].

Writing Eq. (75) with β_g–function given by Eqs. (84, 85) and (88), we have the following RGE for the monopole charge in the two–loop approximation:

$$\frac{dg^2_{run}}{dt} \approx \frac{g^4_{run}}{48\pi^2} + \frac{g^6_{run}}{(16\pi^2)^2}, \tag{105}$$

or

$$\frac{d\log(\tilde{\alpha})}{dt} \approx \frac{\tilde{\alpha}}{12\pi}\left(1 + 3\frac{\tilde{\alpha}}{4\pi}\right). \tag{106}$$

The values (102) for $g^2_{crit} = g^2_{max1,\,2}$ indicate that the contribution of two loops described by the second term of Eq. (105), or Eq. (106), is about 30%, confirming the validity of perturbation theory.

In general, we are able to estimate the validity of the two–loop approximation for all β–functions and γ, calculating the corresponding ratios of the two–loop contributions to

the one–loop contributions at the maxima of curves 1 and 2:

$$\begin{array}{|l|l|}
\hline
\lambda_{crit} = \lambda_{run}^{max1} \approx -13.16 & \lambda_{crit} = \lambda_{run}^{max2} \approx -7.13 \\
g^2_{crit} = g^2_{max1} \approx 18.61 & g^2_{crit} = g^2_{max2} \approx 15.11 \\
\frac{\gamma^{(2)}}{\gamma^{(1)}} \approx -0.0080 & \frac{\gamma^{(2)}}{\gamma^{(1)}} \approx -0.0065 \\
\frac{\beta^{(2)}_{\mu^2}}{\beta^{(1)}_{\mu^2}} \approx -0.0826 & \frac{\beta^{(2)}_{\mu^2}}{\beta^{(1)}_{\mu^2}} \approx -0.0637 \\
\frac{\beta^{(2)}_{\lambda}}{\beta^{(1)}_{\lambda}} \approx 0.1564 & \frac{\beta^{(2)}_{\lambda}}{\beta^{(1)}_{\lambda}} \approx 0.0412 \\
\frac{\beta^{(2)}_{g}}{\beta^{(1)}_{g}} \approx 0.3536 & \frac{\beta^{(2)}_{g}}{\beta^{(1)}_{g}} \approx 0.2871 \\
\hline
\end{array} \qquad (107)$$

Here we see that all ratios are sufficiently small, i.e. all two–loop contributions are small in comparison with one–loop contributions, confirming the validity of perturbation theory in the two–loop approximation, considered in this model. The accuracy of deviation is worse ($\sim 30\%$) for β_g–function. But it is necessary to emphasize that calculating the border curves 1 and 2 of Fig. 18, we have not used RGE (88) for monopole charge: β_g–function is absent in Eq. (99). Therefore, the calculation of g^2_{crit} according to Eq. (101) does not depend on the approximation of β_g–function. The above–mentioned β_g–function appears only in the second order derivative of V_{eff} which is related with the monopole mass m (see Subsection 5.5.).

Eqs. (51) and (104) give the result (58):

$$\alpha^{-1}_{crit} \approx 5. \qquad (108)$$

which is important for the phase transition at the Planck scale predicted by the MPP.

5.4. Approximate universality of the critical coupling constants

The review of all existing results for α_{crit} and $\tilde{\alpha}_{crit}$ gives:

1. $$\alpha^{lat}_{crit} = 0.20 \pm 0.015 \quad \text{and} \quad \tilde{\alpha}^{lat}_{crit} = 1.25 \pm 0.10 \qquad (109)$$

   — in the compact QED with the Wilson lattice action [54];

2. $$\alpha^{lat}_{crit} \approx 0.204, \quad \tilde{\alpha}^{lat}_{crit} \approx 1.25 \qquad (110)$$

— in the model with the Wilson loop action [20];

3.

$$\alpha_{crit} \approx 0.18, \quad \tilde{\alpha}_{crit} \approx 1.36 \tag{111}$$

— in the compact QED with the Villain lattice action [61];

4.

$$\alpha_{crit} = \alpha_A \approx 0.208, \quad \tilde{\alpha}_{crit} = \tilde{\alpha}_A \approx 1.20 \tag{112}$$

— in the HMM [23, 25].

It is necessary to emphasize, that the functions $\alpha(\beta)$ in Fig. 12, describing the effective electric fine structure constant in the vicinity of the phase transition point $\beta_{crit} \approx 1$, are different for the Wilson and Villain lattice actions in the $U(1)$ lattice gauge theory, but the critical values of $\alpha(\beta)$ coincide for both theories [54].

Hereby we see an additional arguments for the previously hoped [8, 20] "approximate universality" of the first order phase transition critical coupling constants: for example, at the phase transition point the fine structure constant α is approximately the same one for various parameters and different regularization schemes.

The most significant conclusion of MPP, which predicts the values of gauge couplings arranging just the MCP, where all phases of the given theory meet, is possibly that the calculations of Refs. [8, 20] suggest the validity of the approximate universality of the critical couplings [16, 19]. It was shown in Refs. [22–29] that one can crudely calculate the phase transition couplings without using any specific lattice, rather only approximating the lattice artifact monopoles as fundamental (point–like) magnetically charged particles condensing. Thus, the details of the lattice — hypercubic or random, with multi–plaquette terms or without them, etc., — also the details of the regularization — lattice or Wilson loops, lattice or the Higgs monopole model — do not matter for values of the phase transition couplings so much. Critical couplings depend only on groups with any regularization. Such an approximate universality is, of course, absolutely needed if there is any sense in relating lattice phase transition couplings to the experimental couplings found in Nature. Otherwise, such a comparison would only make sense if we could guess the true lattice in the right model, what sounds too ambitious.

5.5. Triple point of the HMM phase diagram

In this Section we demonstrate the existence of the triple point on the phase diagram of HMM [23].

Considering the second derivative of the effective potential:

$$V''_{eff}\left(\phi_0^2\right) \equiv \frac{\partial^2 V_{eff}}{\partial (\phi^2)^2}, \tag{113}$$

we can calculate it for the RG improved effective potential (9):

$$\begin{aligned} V''_{eff}\left(\phi^2\right) &= \frac{V'_{eff}\left(\phi^2\right)}{\phi^2} + \left(-\frac{1}{2}\mu^2_{run} + \frac{1}{2}\frac{d^2\mu^2_{run}}{dt^2} + 2\frac{d\mu^2_{run}}{dt}\frac{d\log(G)}{dt} + \mu^2_{run}\frac{d^2\log(G)}{dt^2} \right. \\ &\left. +2\mu^2_{run}\left(\frac{d\log(G)}{dt}\right)^2\right)\frac{G^2}{\phi^2} + \left(\frac{1}{2}\frac{d\lambda_{run}}{dt} + \frac{1}{4}\frac{d^2\lambda_{run}}{dt^2} + 2\frac{d\lambda_{run}}{dt}\frac{d\log(G)}{dt}\right. \\ &\left. +2\lambda_{run}\frac{d\log(G)}{dt} + \lambda_{run}\frac{d^2\log(G)}{dt^2} + 4\lambda_{run}\left(\frac{d\log(G)}{dt}\right)^2\right) G^4(t). \qquad (114) \end{aligned}$$

Let us consider now the case when this second derivative changes its sign giving a maximum of V_{eff} instead of the minimum at $\phi^2 = \phi_0^2$. Such a possibility is shown in Fig. 17 by the dashed curve 2. Now the two additional minima at $\phi^2 = \phi_1^2$ and $\phi^2 = \phi_2^2$ appear in our theory. They correspond to the two different confinement phases for the confinement of electrically charged particles if they exist in the system. When these two minima are degenerate, we have the following requirements:

$$V_{eff}\left(\phi_1^2\right) = V_{eff}\left(\phi_2^2\right) < 0 \quad \text{and} \quad V'_{eff}\left(\phi_1^2\right) = V'_{eff}\left(\phi_2^2\right) = 0, \qquad (115)$$

which describe the border between the confinement phases "Conf. 1" and "Conf. 2" presented in Fig. 19. This border is given as a curve "3" at the phase diagram $(\lambda_{run};\, g^4_{run})$ shown in Fig. 19. The curve "3" meets the curve "1" at the triple point A. According to the illustration of Fig. 19, the triple point A is given by the following requirements:

$$V_{eff}\left(\phi_0^2\right) = V'_{eff}\left(\phi_0^2\right) = V''_{eff}\left(\phi_0^2\right) = 0. \qquad (116)$$

In contrast to the requirements:

$$V_{eff}\left(\phi_0^2\right) = V'_{eff}\left(\phi_0^2\right) = 0, \qquad (117)$$

describing the curve "1", let us consider the joint solution of the following equations:

$$V_{eff}\left(\phi_0^2\right) = V''_{eff}\left(\phi_0^2\right) = 0. \qquad (118)$$

For simplicity, we have considered the one–loop approximation. Using Eqs. (94, 114) and (82–84), it is easy to obtain the solution of Eq. (118) in the one–loop approximation:

$$\mathcal{F}\left(\lambda_{run},\, g^2_{run}\right) = 0, \qquad (119)$$

where

$$\mathcal{F}\left(\lambda_{run}, g^2_{run}\right) = 5g^6_{run} + 24\pi^2 g^4_{run} + 12\lambda_{run} g^4_{run} - 9\lambda^2_{run} g^2_{run} + 36\lambda^3_{run} + 80\pi^2\lambda^2_{run} + 64\pi^4\lambda_{run}. \qquad (120)$$

The dashed curve "2" of Fig. 19 represents the solution of Eq. (119) which is equivalent to Eqs. (118). The curve "2" is going very close to the maximum of the curve "1". Assuming that the position of the triple point A coincides with this maximum let us

consider the border between the phase "Conf. 1", having the first minimum at nonzero ϕ_1 with $V_{eff}^{min}(\phi_1^2) = c_1 < 0$, and the phase "Conf. 2" which reveals two minima with the second minimum being the deeper one and having $V_{eff}^{min}(\phi_2^2) = c_2 < 0$. This border (described by the curve "3" of Fig. 19) was calculated in the vicinity of the triple point A by means of Eq. (115) with ϕ_1 and ϕ_2 represented as $\phi_{1,2} = \phi_0 \pm \epsilon$ with $\epsilon \ll \phi_0$. The result of such calculations gives the following expression for the curve "3":

$$g_{PT}^4 = \frac{5}{2}\left(5\lambda_{run} + 8\pi^2\right)\lambda_{run} + 8\pi^4. \tag{121}$$

The curve "3" meets the curve "1" at the triple point A.

The piece of the curve "1" to the left of the point A describes the border between the "Coulomb–like" phase and phase "Conf. 1". In the vicinity of the triple point A the second derivative $V''_{eff}(\phi_0^2)$ changes its sign leading to the existence of the maximum at $\phi^2 = \phi_0^2$, in correspondence with the dashed curve "2" of Fig. 19. By this reason, the curve "1" of Fig. 19 does not already describe a phase transition border up to the next point B when the curve "2" again intersects the curve "1" at $\lambda_{(B)} \approx -12.24$. This intersection (again giving $V''_{eff}(\phi_0^2) > 0$) occurs surprisingly quickly.

The right piece of the curve "1" along to the right of the point B shown in Fig. 19 separates the "Coulomb" phase and the phase "Conf. 2". But between the points A and B the phase transition border is going slightly upper the curve "1". This deviation is very small and cannot be distinguished on Fig. 19.

It is necessary to note that only $V''_{eff}(\phi^2)$ contains the derivative dg_{run}^2/dt. The joint solution of equations (116) leads to the joint solution of Eqs. (100) and (119). This solution was obtained numerically and gave the following triple point values of λ_{run} and g_{run}^2:

$$\lambda_{(A)} \approx -13.41, \quad g_{(A)}^2 \approx 18.61. \tag{122}$$

The solution (122) demonstrates that the triple point A exists in the very neighbourhood of the maximum of the curve (100). The position of this maximum is given by the following analytical expressions, together with their approximate values:

$$\lambda_{(A)} \approx -\frac{4\pi^2}{3} \approx -13.2, \tag{123}$$

$$g_{(A)}^2 = g_{crit}^2\Big|_{\text{for } \lambda_{run}=\lambda_{(A)}} \approx \frac{4\sqrt{2}}{3}\pi^2 \approx 18.6. \tag{124}$$

Finally, we can conclude that the phase diagram shown in Fig. 19 gives such a description: there exist three phases in the dual sector of the Higgs scalar electrodynamics — the Coulomb–like phase and confinement phases "Conf. 1" and "Conf. 2".

The border "1", which is described by the curve (100), separates the Coulomb–like phase (with $V_{eff} \geq 0$) and confinement phases (with $V_{eff}^{min}(\phi_0^2) < 0$). The curve "1" corresponds to the joint solution of the equations $V_{eff}(\phi_0^2) = V'_{eff}(\phi_0^2) = 0$.

The dashed curve "2" represents the solution of the equations $V_{eff}\left(\phi_0^2\right) = V''_{eff}\left(\phi_0^2\right) = 0$.

The phase border "3" of Fig. 19 separates the two confinement phases. The following requirements take place for this border:

$$V_{eff}\left(\phi_{1,2}^2\right) < 0, \quad V_{eff}\left(\phi_1^2\right) = V_{eff}\left(\phi_2^2\right), \quad V'_{eff}\left(\phi_1^2\right) = V'_{eff}\left(\phi_2^2\right) = 0,$$

$$V''_{eff}\left(\phi_1^2\right) > 0, \quad V''_{eff}\left(\phi_2^2\right) > 0. \tag{125}$$

The triple point A is a boundary point of all three phase transitions shown in the phase diagram of Fig. 19. For $g^2 < g^2_{(A)}$ the field system, described by our model, exists in the confinement phase where all electric charges have to be confined.

Taking into account that monopole mass m is given by the following expression:

$$V''_{eff}\left(\phi_0^2\right) = \frac{1}{4\phi_{0A}^2} \frac{d^2 V_{eff}}{d\phi^2}\bigg|_{\phi=\phi_0} = \frac{m^2}{4\phi_{0A}^2}, \tag{126}$$

we see that monopoles acquire zero mass in the vicinity of the triple point A:

$$V''_{eff}\left(\phi_{0A}^2\right) = \frac{m^2_{(A)}}{4\phi_{0A}^2} = 0. \tag{127}$$

This result is in agreement with the result of the compact QED [77]: $m^2 \to 0$ in the vicinity of the critical point.

6. "ANO–strings", or the vortex description of the confinement phases

As it was shown in the previous Subsection, two regions between the curves "1", "3" and "3", "1", given by the phase diagram of Fig. 19, correspond to the existence of the two confinement phases, different in the sense that the phase "Conf. 1" is produced by the second minimum, but the phase "Conf. 2" corresponds to the third minimum of the effective potential. It is obvious that in this case both phases have nonzero monopole condensate in the minima of the effective potential, when $V^{min}_{eff}\left(\phi_{1,2} \neq 0\right) < 0$. By this reason, the Abrikosov–Nielsen–Olesen (ANO) electric vortices (see Refs. [78, 79]) may exist in these both phases, which are equivalent in the sense of the "string" formation. If electric charges are present in a model (they are absent in HMM), then these charges are placed at the ends of the vortices–"strings" and therefore are confined. But only closed "strings" exist in the confinement phases of HMM. The properties of the "ANO–strings" in the $U(1)$ gauge theory were investigated in Ref. [23].

In the London's limit ($\lambda \to \infty$) the dual Abelian Higgs model developed in Refs. [78, 79] and described by the Lagrangian (62), gives the formation of monopole condensate with amplitude ϕ_0, which repels and suppresses the electromagnetic field $F_{\mu\nu}$ almost everywhere, except the region around the vortex lines. In this limit, we have the following London equation:

$$\text{rot}\ \vec{j}^m = \delta^{-2}\vec{E}, \tag{128}$$

where $\vec{j}^m$ is the microscopic current of monopoles, $\vec{E}$ is the electric field strength and δ is the penetration depth. It is clear that δ^{-1} is the photon mass m_V, generated by the Higgs mechanism. The closed equation for $\vec{E}$ follows from the Maxwell equations and Eq. (128) just in the London's limit.

In our case δ is defined by the following relation:

$$\delta^{-2} \equiv m_V^2 = g^2\phi_0^2. \tag{129}$$

On the other hand, the field ϕ has its own correlation length ξ, connected to the mass of the field ϕ ("the Higgs mass"):

$$\xi = {m_S}^{-1}, \quad m_S^2 = \lambda\phi_0^2. \tag{130}$$

The London's limit for our "dual superconductor of the second type" corresponds to the following relations:

$$\delta \gg \xi, \quad m_V \ll m_S, \quad g \ll \lambda, \tag{131}$$

and "the string tension" — the vortex energy per unit length [78] — is (for the minimal electric vortex flux 2π):

$$\sigma = \frac{2\pi}{g^2\delta^2}\ln\left(\frac{\delta}{\xi}\right) = 2\pi\phi_0^2\ln\left(\frac{m_S}{m_V}\right), \quad \text{where} \quad \frac{\delta}{\xi} = \frac{m_S}{m_V} \gg 1. \tag{132}$$

We see that the ANO–theory in the London's limit implies the photon mass generation: $m_V = 1/\delta$, which is much less than the Higgs mass $m_S = 1/\xi$.

Let us wonder now, whether our "strings" are thin or thick.

The vortex may be considered as thin, if the distance L between the electric charges sitting at its ends, i.e. the string length, is much larger than the penetration length δ:

$$L \gg \delta \gg \xi. \tag{133}$$

It is obvious that only rotating "strings" can exist as stable states. In the framework of classical calculations, it is not difficult to obtain the mass M and angular momentum J of the rotating "string":

$$J = \frac{1}{2\pi\sigma}M^2, \quad M = \frac{\pi}{2}\sigma L. \tag{134}$$

The following relation follows from Eq. (134):

$$L = 2\sqrt{\frac{2J}{\pi\sigma}}, \tag{135}$$

or

$$L = \frac{2g\delta}{\pi}\sqrt{\frac{J}{\ln\left(\frac{m_S}{m_V}\right)}}. \tag{136}$$

For $J = 1$ we have:

$$\frac{L}{\delta} = \frac{2g}{\pi\sqrt{\ln\left(\frac{m_S}{m_V}\right)}}, \tag{137}$$

what means that for $m_S \gg m_V$ the length of this "string" is small and does not obey the requirement (133). It is easy to see from Eq. (136) that in the London's limit the "strings" are very thin ($L/\delta \gg 1$) only for the enormously large angular momenta $J \gg 1$.

The phase diagram of Fig. 19 shows the existence of the confinement phase for $\alpha \geq \alpha_{(A)}$. This means that the formation of (closed) vortices begins at the triple point $\alpha = \alpha_{(A)}$: for $\alpha > \alpha_{(A)}$, i.e. $\tilde{\alpha} < \tilde{\alpha}_{(A)}$, we have nonzero ϕ_0 leading to the creation of vortices.

In Section 4.4. we have shown that the lattice investigations lead to the "freezing" of the electric fine structure constant at the value $\alpha = \alpha_{max}$ and mentioned that the authors of Ref. [65] predicted: $\alpha_{max} = \pi/12 \approx 0.26$.

Let us estimate now the region of values of the magnetic charge g in the confinement phase considered in this paper:

$$\begin{aligned} g_{min} &\leq g \leq g_{max}, \\ g_{max} &= g_{(A)} \approx \sqrt{15.1} \approx 3.9, \\ g_{min} &= \sqrt{\frac{\pi}{\alpha_{max}}} \approx 3.5. \end{aligned} \tag{138}$$

Then for $m_S = 10 m_V$ (considered as an example) we have from Eq. (137) the following estimate of the "string" length when $J = 1$:

$$1.5 \lesssim \frac{L}{\delta} \lesssim 1.8. \tag{139}$$

We see that in the $U(1)$ gauge theory the low-lying states of "strings" correspond to the short and thick vortices.

In general, the way of receiving of the Nambu–Goto strings from the dual Abelian Higgs model of scalar monopoles was demonstrated in Ref. [80].

7. Phase transition couplings in the regularized $SU(N)$ gauge theories

It was shown in a lot of investigations (see for example, [58,59] and references there) that the confinement in the $SU(N)$ lattice gauge theories effectively comes to the same $U(1)$

formalism. The reason is the Abelian dominance in the monopole vacuum: monopoles of the Yang–Mills theory are the solutions of the $U(1)$–subgroups, arbitrary embedded into the $SU(N)$ group. After a partial gauge fixing (Abelian projection by 't Hooft [81]) $SU(N)$ gauge theory is reduced to the Abelian $U(1)^{N-1}$ theory with $N-1$ different types of Abelian monopoles. Choosing the Abelian gauge for dual gluons, it is possible to describe the confinement in the lattice $SU(N)$ gauge theories by the analogous dual Abelian Higgs model of scalar monopoles.

7.1. The "abelization" of monopole vacuum in the non–Abelian theories

A lattice imitates the non–perturbative vacuum of zero temperature $SU(2)$ and $SU(3)$ gluodynamics as a condensate of monopoles which emerge as leading non–perturbative fluctuations of the non–Abelian $SU(N)$ gauge theories in the gauge of the Abelian projections by G. 't Hooft [81] (see also the review [82] and Refs. [83–85]). It is possible to find such a gauge, in which monopole degrees of freedom, hidden in the given field configuration, become explicit.

Let us consider the $SU(N)$ gluodynamics. For any composite operator $\mathbf{X} \in$ the adjoint representation of $SU(N)$ group ($\mathbf{X}$ may be $(F_{\mu\nu})_{ij}$, where i, $j = 1, 2, ..., N$) we can find such a gauge:

$$\mathbf{X} \to \mathbf{X'} = \mathbf{V X V^{-1}}, \tag{140}$$

where the unitary matrix $\mathbf{V}$ transforms $\mathbf{X}$ to diagonal $\mathbf{X'}$:

$$\mathbf{X} \to \mathbf{X'} = \mathbf{V X V^{-1}} = \mathrm{diag}\left(\lambda_\mathbf{1}, \lambda_\mathbf{2}, ..., \lambda_\mathbf{N}\right). \tag{141}$$

We can choose the ordering of λ_i:

$$\lambda_1 \leq \lambda_2 \leq ... \leq \lambda_N. \tag{142}$$

The matrix $\mathbf{X'}$ belongs to the Cartain, or Maximal Abelian subgroup of the $SU(N)$ group:

$$U(1)^{N-1} \in SU(N). \tag{143}$$

Let us consider the field A_μ in the diagonal gauge:

$$\bar{A}_\mu = V\left(A_\mu + \frac{i}{g}\partial_\mu\right)V^{-1}. \tag{144}$$

This field transforms according to the subgroup $U(1)^{N-1}$: its diagonal elements

$$\left(a_\mu\right)_i \equiv \left(\bar{A}_\mu\right)_{ii}$$

transform as Abelian gauge fields (photons):

$$\left(a_\mu\right)_i \to \left(a'_\mu\right)_i = \left(a_\mu\right)_i + \frac{1}{g}\partial_\mu\alpha_i, \tag{145}$$

but its non–diagonal elements

$$(c_\mu)_{ij} \equiv \left(\bar{A}_\mu\right)_{ij} \quad \text{with} \quad i \neq j$$

transform as charged fields:

$$\left(c'_\mu\right)_{ij} = \exp\left[i\left(\alpha_i - \alpha_j\right)\right](c_\mu)_{ij}, \tag{146}$$

where i, $j = 1$, 2, ..., N.

According to G.'t Hooft [81], if some λ_i coincide, then the singularities, having the properties of monopoles, appear in the "Abelian part" of the non–Abelian gauge fields. Indeed, let us consider the strength tensor of the "Abelian gluons":

$$\begin{aligned}(f_{\mu\nu})_i &= \partial_\mu(a_\nu)_i - \partial_\nu(a_\mu)_i \\ &= VF_{\mu\nu}V^{-1} + ig\left[V\left(A_\mu + \frac{i}{g}\partial_\mu\right)V^{-1}, V\left(A_\nu + \frac{i}{g}\partial_\nu\right)V^{-1}\right].\end{aligned} \tag{147}$$

The monopole current is:

$$(K_\mu)_i = \frac{1}{8\pi}\epsilon_{\mu\nu\rho\sigma}\partial_\nu(f_{\rho\sigma})_i, \tag{148}$$

and it is conserved:

$$\partial_\mu(K_\mu)_i = 0. \tag{149}$$

$F_{\mu\nu}$ had no singularities. Therefore, all singularities can come from the commutator, which is written in Eq. (147).

The magnetic charge $m_i(\Omega)$ in 3d–volume Ω is:

$$m_i(\Omega) = \int_\Omega d^3\sigma_\mu(K_\mu)_i = \frac{1}{8\pi}\int_{\partial\Omega} d^2\sigma_{\mu\nu}(f_{\mu\nu})_i. \tag{150}$$

If $\lambda_1 = \lambda_2$ (coincide) at the point $x^{(1)}$ in 3d–volume Ω, then we have a singularity on the curve in 4d–space, which is a world–line of the magnetic monopole, and $x = x^{(1)}$ is a singular point of the gauge transformed fields $\bar{A}_\mu$ and $(a_\mu)_i$.

As it was shown by 't Hooft [81]:

$$(f_{\mu\nu})_i \sim O\left(\left|x - x^{(1)}\right|^{-2}\right) \tag{151}$$

only in the vicinity of $x^{(1)}$, where it behaves as a magnetic field of the point–like monopole.

Finally, we have the following conclusions:

1. The initial potentials A_μ and strength tensor $F_{\mu\nu}$ had no singularities.

2. At large distances $(f_{\mu\nu})_i$ doesn't have a behaviour

   $$(f_{\mu\nu})_i \sim O\left(\left|x - x^{(1)}\right|^{-2}\right)$$

   and the monopoles exist only near $x = x^{(1)}$.

3. Fields $\bar{A}_\mu$ and $(a_\mu)_i$ are not classical solutions: they are a result of the quantum fluctuations of gluon fields.

4. Any distribution of gluon fields in the vacuum can undergo the Abelian projection.

We have seen that in the $SU(N)$ gauge theories quantum fluctuations (non–perturbative effects) of gluon fields reveal an Abelian vacuum monopoles and suppress the non–diagonal components of the strength tensor $(F_{\mu\nu})_{ij}$. As it will be shown below, this phenomenon gives very important consequences for the Planck scale physics.

Using the idea of the monopole vacuum "abelization" of the $SU(N)$ lattice gauge theories, a method of theoretical estimate of the $SU(N)$ critical couplings was developed in Ref. [24].

7.2. Monopoles strength group dependence

Lattice non–Abelian gauge theories also have lattice artifact monopoles. It was supposed in Ref. [24] that only those lattice artifact monopoles are important for the phase transition calculations which have the smallest monopole charges.

Let us consider the lattice gauge theory with the gauge group $SU(N)/Z_N$ as a main example. That is to say, we consider the adjoint representation action and do not distinguish link variables forming the same one multiplied by any element of the center of the group. The group $SU(N)/Z_N$ is not simply connected and has the first homotopic group $\Pi_1(SU(N)/Z_N)$ equal to Z_N. The lattice artifact monopole with the smallest magnetic charge may be described as a three–cube (or rather a chain of three–cubes describing the time track) from which radiates magnetic field corresponding to the $U(1)$ subgroup of the gauge group $SU(N)/Z_N$ with the shortest length insight of this group, but still homotopically non–trivial. In fact, this $U(1)$ subgroup is obtained by the exponentiating generator:

$$\frac{\text{``}\lambda_8\text{''}}{2} = \frac{1}{\sqrt{2N(N-1)}} \begin{pmatrix} N-1 & 0 & \cdots & 0 \\ 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 \end{pmatrix}. \qquad (152)$$

This specific form is one gauge choice; any similarity transformation of this generator would describe physically the same monopole. If one has somehow already chosen the gauge monopoles with different but similarity transformation related generators, they would be physically different. Thus, after gauge choice, there are monopoles corresponding to different directions of the Lie algebra generators in the form $\mathcal{U}\frac{\text{``}\lambda_8\text{''}}{2}\mathcal{U}^+$.

Now, when we want to apply the effective potential calculation as a technique for the getting phase diagram information for the condensation of the lattice artifact monopoles

in the non–Abelian lattice gauge theory, we have to correct the Abelian case calculation for the fact that after gauge choice we have a lot of different monopoles. If a couple of monopoles happens to have their generators just in the same directions in the Lie algebra, they will interact with each other as Abelian monopoles (in first approximation). In general, the interaction of two monopoles by exchange of a photon will be modified by the following factor:

$$\frac{\mathrm{Tr}\left(\mathcal{U}_1\frac{``\lambda_8"}{2}\mathcal{U}_1^{+}\mathcal{U}_2\frac{``\lambda_8"}{2}\mathcal{U}_2^{+}\right)}{\mathrm{Tr}\left(\frac{``\lambda_8"}{2}\right)^2}. \tag{153}$$

We shall assume that we can correct these values of monopole orientations in the Lie algebra in a statistical way. That is to say, we want to determine an effective coupling constant $\tilde{g}_{eff}$ describing the monopole charge as if there is only one Lie algebra orientation–wise type of monopole. It should be estimated statistically in terms of the magnetic charge $\tilde{g}_{\text{genuine}}$ valid to describe the interaction between monopoles with generators oriented along the same $U(1)$ subgroup. A very crude intuitive estimate of the relation between these two monopole charge concepts $\tilde{g}_{\text{genuine}}$ and $\tilde{g}_{eff}$ consists in playing that the generators are randomly oriented in the whole N^2-1 dimensional Lie algebra. When even the sign of the Lie algebra generator associated with the monopole is random — as we assumed in this crude argument — the interaction between two monopoles with just one photon exchanged averages out to zero. Therefore, we can get a non–zero result only in the case of exchange by two photons or more. That is, however, good enough for our effective potential calculation since only $\tilde{g}^4$ (but not the second power) occurs in the Coleman–Weinberg effective potential in the one–loop approximation (see [36, 37]). Taking into account this fact that we can average imagining monopoles with generators along a basis vector in the Lie algebra, the chance of interaction by double photon exchange between two different monopoles is just $1/(N^2-1)$, because there are N^2-1 basis vectors in the basis of the Lie algebra. Thus, this crude approximation gives:

$$\tilde{g}^4_{eff} = \frac{1}{N^2-1}\tilde{g}^4_{\text{genuine}}. \tag{154}$$

Note that considering the two photons exchange which is forced by our statistical description, we must concern the fourth power of the monopole charge $\tilde{g}$.

The relation (154) was not derived correctly, but its validity can be confirmed if we use a more correct statistical argument. The problem with our crude estimate is that the generators making monopole charge to be minimal must go along the shortest type of $U(1)$ subgroups with non–trivial homotopy.

Correct averaging

The λ_8–like generators $\mathcal{U}\frac{``\lambda_8"}{2}\mathcal{U}^{+}$ maybe written as

$$\mathcal{U}\frac{``\lambda_8"}{2}\mathcal{U}^{+} = -\sqrt{\frac{1}{2N(N-1)}}\mathbf{1} + \sqrt{\frac{N}{2(N-1)}}\mathcal{P}, \tag{155}$$

where $\mathcal{P}$ is a projection metrics into one–dimensional state in the $\underline{\mathrm{N}}$ representation. It is easy to see that averaging according to the Haar measure distribution of $\mathcal{U}$, we get the average of $\mathcal{P}$ projection on "quark" states with a distribution corresponding to the rotationally invariant one on the unit sphere in the N–dimensional $\underline{\mathrm{N}}$–Hilbert space.

If we denote the Hilbert vector describing the state on which $\mathcal{P}$ shall project as

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{pmatrix}, \tag{156}$$

then the probability distribution on the unit sphere becomes:

$$P\left(\begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{pmatrix}\right)\prod\nolimits_{i=1}^{N} d\psi_i \propto \delta\left(\sum\nolimits_{i=1}^{N} |\psi_i|^2 - 1\right)\prod_{i=1}^{N} d\left(|\psi_i|^2\right). \tag{157}$$

Since, of course, we must have $|\psi_i|^2 \geq 0$ for all $i = 1,\, 2,\, ...,\, N$, the δ–function is easily seen to select a flat distribution on a $(N-1)$–dimensional equilateral simplex. The average of the two photon exchange interaction given by the correction factor (153) squared (numerically):

$$\frac{\mathrm{Tr}\left(\mathcal{U}_1 \frac{\text{“}\lambda_8\text{”}}{2} \mathcal{U}_1^{+} \mathcal{U}_2 \frac{\text{“}\lambda_8\text{”}}{2} \mathcal{U}_2^{+}\right)^2}{\mathrm{Tr}\left(\left(\frac{\text{“}\lambda_8\text{”}}{2}\right)^2\right)^2} \tag{158}$$

can obviously be replaced by the expression where we take as random only one of the "random" λ_8–like generators, while the other one is just taken as $\frac{\text{“}\lambda_8\text{”}}{2}$, i.e. we can take say $\mathcal{U}_2 = \mathbf{1}$ without changing the average.

Considering the two photon exchange diagram, we can write the correction factor (obtained by the averaging) for the fourth power of magnetic charge:

$$\frac{\tilde{g}^4_{eff}}{\tilde{g}^4_{\text{genuine}}} = \text{average}\left\{\frac{\mathrm{Tr}\left(\frac{\text{“}\lambda_8\text{”}}{2} \mathcal{U}_1 \frac{\text{“}\lambda_8\text{”}}{2} {\mathcal{U}_1}^{+}\right)^2}{\mathrm{Tr}\left(\left(\frac{\text{“}\lambda_8\text{”}}{2}\right)^2\right)^2}\right\}. \tag{159}$$

Substituting the expression (155) in Eq. (159), we have:

$$\frac{\tilde{g}^4_{eff}}{\tilde{g}^4_{\text{genuine}}} = \text{average}\left\{\frac{\mathrm{Tr}\left(\frac{\text{“}\lambda_8\text{”}}{2}\left(-\sqrt{\frac{1}{2N(N-1)}}\mathbf{1} + \sqrt{\frac{N}{2(N-1)}}\mathcal{P}\right)\right)^2}{\mathrm{Tr}\left(\left(\frac{\text{“}\lambda_8\text{”}}{2}\right)^2\right)^2}\right\}. \tag{160}$$

Since $\frac{\text{“}\lambda_8\text{”}}{2}$ is traceless, we obtain using the projection (156):

$$\mathrm{Tr}\left(\frac{\text{“}\lambda_8\text{”}}{2}\mathcal{P}\sqrt{\frac{N}{2(N-1)}}\right) = -\frac{1}{2(N-1)} + \frac{N}{2(N-1)}|\psi_1|^2. \tag{161}$$

The value of the square $|\psi_1|^2$ over the simplex is proportional to one of the heights in this simplex. It is obvious from the geometry of a simplex that the distribution of $|\psi_1|^2$ is

$$dP = (N-1)\left(1-|\psi_1|^2\right)^{(N-2)} d\left(|\psi_1|^2\right), \tag{162}$$

where, of course, $0 \le |\psi_1|^2 \le 1$ only is allowed. In Eq. (162) the quantity P is a probability.

By definition:

$$\text{average}\left\{f(|\psi_1|^2)\right\} = (N-1)\int_0^1 f\left(|\psi_1|^2\right)\left(1-|\psi_1|^2\right)^{(N-2)} d\left(|\psi_1|^2\right). \tag{163}$$

Then

$$\begin{aligned}
\frac{\tilde{g}^4_{eff}}{\tilde{g}^4_{\text{genuine}}} &= \frac{N^2}{(N-1)}\int_0^1 \left(\frac{1}{N}-|\psi_1|^2\right)^2 \left(1-|\psi_1|^2\right)^{N-2} d\left(|\psi_1|^2\right) && (164)\\
&= \frac{N^2}{N-1}\int_0^1 \left(1-y-\frac{1}{N}\right)^2 y^{N-2} dy && (165)\\
&= \frac{1}{N^2-1} && (166)
\end{aligned}$$

and we have confirmed our crude estimate (154).

Relative normalization of couplings

Now we are interested in how $\tilde{g}^2_{\text{genuine}}$ is related to $\alpha_N = g_N^2/4\pi$.

We would get the simple Dirac relation:

$$g_{(1)} \cdot \tilde{g}_{\text{genuine}} = 2\pi, \tag{167}$$

if $g_{(1)} \equiv g_{U(1)-\text{subgroup}}$ is the coupling for the $U(1)$–subgroup of $SU(N)$ normalized in such a way that the charge quantum $g_{(1)}$ corresponds to a covariant derivative $\partial_\mu - g_{(1)} A_\mu^{U(1)}$.

Then we shall follow the convention — usually used to define $\alpha_N = g_N^2/4\pi$ — that the covariant derivative for the $\underline{\text{N}}$–plet representation is:

$$D_\mu = \partial_\mu - g_N \frac{\lambda^a}{2} A_\mu^a \tag{168}$$

with

$$Tr\left(\frac{\lambda^a}{2}\frac{\lambda^b}{2}\right) = \frac{1}{2}\delta^{ab}, \tag{169}$$

and the gauge field kinetic term is

$$L = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}, \tag{170}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_N f^{abc} A^b_\mu A^c_\nu. \tag{171}$$

Especially if we want to choose a basis for our generalized Gell–Mann matrices so that one basic vector is our $\frac{\text{“}\lambda_8\text{”}}{2}$, then for $A_\mu^{\text{“}8\text{”}}$ we have the covariant derivation $\partial_\mu - g_N \frac{\text{“}\lambda_8\text{”}}{2} A_\mu^{\text{“}8\text{”}}$. If this covariant derivative is written in terms of the $U(1)$–subgroup, corresponding to monopoles with the Dirac relation (167), then the covariant derivative has a form $\partial_\mu - g_{(1)} A_\mu^8 \cdot \underline{\underline{M}}$. Here $\underline{\underline{M}}$ has the property that $\exp\left(i2\pi \underline{\underline{M}}\right)$ corresponds to the elements of the group $SU(N)/Z_N$ going all around and back to the unit element. Of course, $\underline{\underline{M}} = \frac{g_N}{g_{(1)}} \cdot \frac{\text{“}\lambda_8\text{”}}{2}$ and the ratio $g_N/g_{(1)}$ must be such one that $\exp\left(i2\pi \frac{g_N}{g_{(1)}} \frac{\text{“}\lambda_8\text{”}}{2}\right)$ shall represent — after first return — the unit element of the group $SU(N)/Z_N$. Now this unit element really means the coset consisting of the center elements $\exp\left(i\frac{2\pi k}{N}\right) \in SU(N)$, $(k \in Z)$, and the requirement of the normalization of $g_{(1)}$ ensuring the Dirac relation (167) is:

$$\exp\left(i2\pi \frac{g_N}{g_{(1)}} \frac{\text{“}\lambda_8\text{”}}{2}\right) = \exp\left(i\frac{2\pi}{N}\right) \mathbf{1}. \tag{172}$$

This requirement is satisfied if the eigenvalues of $\frac{g_N}{g_{(1)}} \frac{\text{“}\lambda_8\text{”}}{2}$ are modulo 1 equal to $-1/N$, i.e. formally we might write:

$$\frac{g_N}{g_{(1)}} \frac{\text{“}\lambda_8\text{”}}{2} = -\frac{1}{N} \pmod{1}. \tag{173}$$

According to (152), we have:

$$\frac{g_N}{g_{(1)}} \cdot \frac{1}{\sqrt{2N(N-1)}} \begin{pmatrix} N-1 & 0 & \cdots & 0 \\ 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 \end{pmatrix} = -\frac{1}{N} \pmod{1}, \tag{174}$$

what implies:

$$\frac{g_N}{g_{(1)}} = \sqrt{\frac{2(N-1)}{N}}, \tag{175}$$

or

$$\frac{g_N^2}{g_{(1)}^2} = \frac{2(N-1)}{N}. \tag{176}$$

7.3. The relation between $U(1)$ and $SU(N)$ critical couplings

Collecting the relations (167, 176) and (154), we get:

$$\begin{aligned} \alpha_N^{-1} &= \frac{4\pi}{g_N^2} = \frac{N}{2(N-1)} \cdot \frac{4\pi}{g_{(1)}^2} = \frac{N}{2(N-1)} \cdot \frac{\tilde{g}^2_{\text{genuine}}}{\pi} \\ &= \frac{N}{2(N-1)} \sqrt{N^2-1} \cdot \frac{\tilde{g}^2_{eff}}{\pi} = \frac{N}{2(N-1)} \sqrt{N^2-1} \cdot \frac{4\pi}{g^2_{U(1)}} \\ &= \frac{N}{2} \sqrt{\frac{N+1}{N-1}} \cdot \alpha^{-1}_{U(1)}, \end{aligned} \tag{177}$$

where

$$g_{U(1)}\tilde{g}_{eff} = 2\pi \tag{178}$$

and $\alpha_{U(1)} = g^2_{U(1)}/4\pi$.

The meaning of this result is that provided that we have $\tilde{g}_{eff}$ the same for $SU(N)/Z_N$ and $U(1)$ gauge theories the couplings are related according to Eq. (177).

We have a use for this relation when we want to calculate the phase transition couplings considering the scalar monopole field responsible for the phase transition in the gauge groups $SU(N)/Z_N$. Having in mind the "Abelian" dominance in the $SU(N)$ monopole vacuum, we must think that $\tilde{g}^{crit}_{eff}$ coincides with g_{crit} of the $U(1)$ gauge theory. Of course, here we have an approximation taking into account only monopoles interaction and ignoring the relatively small self–interactions of the Yang–Mills fields. In this approximation we obtain the same phase transition (triple point, or critical) $\tilde{g}_{eff}$–coupling which is equal to g_{crit} of $U(1)$ whatever the gauge group $SU(N)$ might be. Thus we conclude that for the various groups $U(1)$ and $SU(N)/Z_N$, according to Eq. (177), we have the following relation between the phase transition couplings:

$$\alpha^{-1}_{N,\,crit} = \frac{N}{2}\sqrt{\frac{N+1}{N-1}}\alpha^{-1}_{U(1),\,crit}. \tag{179}$$

Using the relation (179), we obtain:

$$\alpha^{-1}_{U(1),\,crit} : \alpha^{-1}_{2,\,crit} : \alpha^{-1}_{3,\,crit} = 1 : \sqrt{3} : \frac{3}{\sqrt{2}} = 1 : 1.73 : 2.12. \tag{180}$$

These relations are used below for the explanation of predictions of the MPP.

8. G–theory, or Anti–grand unification theory (AGUT)

Having an interest in the fundamental laws of physics, we can consider the two possibilities:

1. At very small (Planck length) distances *our space–time is continuous* and there exists the fundamental theory (maybe with a very high symmetry) which we do not know at present time.

2. At very small distances *our space–time is discrete*, and this discreteness influences on the Planck scale physics.

The theory of Scale Relativity (SR) [86,87] predicts that there exists a minimal scale of the space–time resolution equal to the Planck length λ_P, which can be considered as a

fundamental scale of our Nature. This gives us a reason to make an assumption that *our* $(3+1)$*–dimensional space is discrete on the fundamental level.*

This may be an initial (basic) point of view of the theory, which takes a discreteness as existing, not as the lattice computation trick in QCD, say. In the simplest case we can imagine our $(3+1)$ space–time as a regular hypercubic lattice with a parameter $a = \lambda_P$. Then the lattice artifact monopoles can play an essential role near the Planck scale. But of course, it is necessary to comment that we do not know (at least, on the level of our today knowledge), what lattice–like structure (random lattice, or foam, or string lattice, etc.) is realized in the description of physical processes at very small distances even if there should be a lattice.

Investigating the phase transition in the dual Higgs monopole model, we have pursued two objects. From one side, we had an aim to explain the lattice results. But we had also another aim.

According to the Multiple Point Model (MPM), at the Planck scale there exists a multiple critical point (MCP), which is a boundary point of the phase transitions in $U(1)$, $SU(2)$ and $SU(3)$ sectors of the fundamental regularized gauge theory G. It is natural to assume that the objects responsible for these transitions are the physically existing Higgs scalar monopoles, which have to be introduced into the theory as fundamental fields. Our calculations indicate that the corresponding critical couplings coincide with the lattice ones, confirming the idea of Ref. [8].

The results reviewed in the present paper are very encouraging for the Anti–Grand Unification Theory (AGUT), which always is used in conjunction with the MPM.

Most efforts to explain the Standard Model (SM) describing well all experimental results known today are devoted to Grand Unification Theories (GUTs). The supersymmetric extension of the SM consists of taking the SM and adding the corresponding supersymmetric partners [88]. The Minimal Supersymmetric Standard Model (MSSM) shows [89] the possibility of the existence of the grand unification point at

$$\mu_{\rm GUT} \sim 10^{16}\ {\rm GeV}.$$

Unfortunately, at present time the experiment does not indicate any manifestation of Supersymmetry. In this connection, the Anti–Grand Unification Theory (AGUT) was developed in Refs. [11], [15–19] as an alternative to SUSY GUTs. According to this theory, supersymmetry does not come into the existence up to the Planck energy scale:

$$M_{Pl} \approx 1.22 \cdot 10^{19}\,{\rm GeV}. \tag{181}$$

The Standard Model (SM) is based on the group SMG described by Eq. (1). AGUT suggests that at the Planck scale: $\mu_G \sim \mu_{Pl} = M_{Pl}$ there exists the more fundamental group G containing a number of copies of the Standard Model Group SMG.

9. Family Replicated Gauge Group Model (FRGGM) as an extension of the SM

The extension of the Standard Model with the Family Replicated Gauge Group :

$$G = (SMG)^{N_{fam}} = [SU(3)_c]^{N_{fam}} \times [SU(2)_L]^{N_{fam}} \times [U(1)_Y]^{N_{fam}} \tag{182}$$

was first suggested in the paper [11] and developed in the book [12] (see also the review [13]). Here N_{fam} designates the number of quark and lepton families.

If $N_{fam} = 3$ (as our theory predicts and experiment confirms), then the fundamental gauge group G is:

$$G = (SMG)^3 = SMG_{1st\ fam.} \times SMG_{2nd\ fam.} \times SMG_{3rd\ fam.}, \tag{183}$$

or

$$G = (SMG)^3 = [SU(3)_c]^3 \times [SU(2)_L]^3 \times [U(1)_Y]^3. \tag{184}$$

The generalized fundamental group:

$$G_f = (SMG)^3 \times U(1)_f \tag{185}$$

was suggested by fitting the SM charged fermion masses and mixing angles in papers [15, 17].

A new generalization of our FRGG–model was suggested in paper [18], where:

$$\begin{aligned} G_{ext} &= \left(SMG \times U(1)_{(B-L)}\right)^3 \\ &\equiv [SU(3)_c]^3 \times [SU(2)_L]^3 \times [U(1)_Y]^3 \times \left[U(1)_{(B-L)}\right]^3 \end{aligned} \tag{186}$$

is the fundamental gauge group, which takes right–handed neutrinos and the see–saw mechanism into account. This extended model can describe all modern neutrino experiments, giving a reasonable fit to all the quark-lepton masses and mixing angles.

The gauge group $G = G_{ext}$ contains: $3 \times 8 = 24$ gluons, $3 \times 3 = 9$ W–bosons, and $3 \times 1 + 3 \times 1 = 6$ Abelian gauge bosons.

At first sight, this $\left(SMG \times U(1)_{(B-L)}\right)^3$ group with its 39 generators seems to be just one among many possible SM gauge group extensions. However, it is not such an arbitrary choice. There are at least reasonable requirements (postulates) on the gauge group G (or G_f, or G_{ext}) which have uniquely to specify this group. It should obey the following postulates (the first two are also valid for $SU(5)$ GUT):

1. G or G_f should only contain transformations, transforming the known 45 Weyl fermions (= 3 generations of 15 Weyl particles each) — counted as left handed, say — into each other unitarily, so that G (or G_f) must be a subgroup of $U(45)$: $G \subseteq U(45)$.

2. No anomalies, neither gauge nor mixed. AGUT assumes that only straightforward anomaly cancellation takes place and forbids the Green–Schwarz type anomaly cancellation [90].

3. AGUT should NOT UNIFY the irreducible representations under the SM gauge group, called here SMG (see Eq. (1)).

4. G is the maximal group satisfying the above–mentioned postulates.

There are five Higgs fields named ϕ_{WS}, S, W, T, ξ in AGUT extended by Froggatt and Nielsen [15] with the group of symmetry G_f given by Eq. (185). These fields break AGUT to the SM what means that their vacuum expectation values (VEV) are active. The field ϕ_{WS} corresponds to the Weinberg–Salam theory, $\langle S\rangle = 1$, so that we have only three free parameters — three VEVs $\langle W\rangle$, $\langle T\rangle$ and $\langle \xi\rangle$ to fit the experiment in the framework of this model. The authors of Ref. [15] used them with aim to find the best fit to conventional experimental data for all fermion masses and mixing angles in the SM (see Table 1).

The result is encouraging. The fit is given by the χ^2 function (called here $\tilde{\chi}^2$). The lowest value of $\tilde{\chi}^2(\approx 1.87)$ gives the following VEVs:

$$\langle S\rangle = 1; \quad \langle W\rangle = 0.179; \quad \langle T\rangle = 0.071; \quad \langle \xi\rangle = 0.099. \tag{187}$$

The extended AGUT by Nielsen and Takanishi [18], having the group of symmetry G_{ext} (see Eq. (186)), was suggested with aim to explain the neutrino oscillations. Introducing the right–handed neutrino in the model, the authors replaced the assumption 1 and considered $U(48)$ group instead of $U(45)$, so that G_{ext} is a subgroup of $U(48)$: $G_{ext} \subseteq U(48)$. This group ends up having 7 Higgs fields falling into 4 classes according to the order of magnitude of the expectation values:

1. The smallest VEV Higgs field plays role of the SM Weinberg–Salam Higgs field ϕ_{WS} having the weak scale value $\langle\phi_{WS}\rangle = 246\,\text{GeV}/\sqrt{2}$.

2. The next smallest VEV Higgs field breaks all families $U(1)_{(B-L)}$ group, which is broken at the see–saw scale. This VEV is $\langle\phi_{(B-L)}\rangle \sim 10^{12}$ GeV. Such a field is absent in the "old" extended AGUT.

3. The next 4 Higgs fields are W, T, ξ and χ, which have VEVs of the order of a factor 10 to 50 under the Planck unit. That means that if intermediate propagators have scales given by the Planck scale, as it is assumed in AGUT in general, then they will give rise to suppression factors of the order 1/10 each time they are needed to cause a transition. The field χ is absent in the "old" G_f–AGUT. It was introduced in Refs. [17, 18] for the purpose of the study of neutrinos.

4. The last one, with VEV of the same order as the Planck scale, is the Higgs field S. It had VEV $\langle S\rangle = 1$ in the "old" extended AGUT [15] by Froggatt and Nielsen (with G_f group of symmetry), but this VEV is not equal to unity in the "new" extended AGUT [18]. Therefore there is a possibility to observe phenomenological consequences of the field S in the Nielsen–Takanishi model [18].

In contrast to the "old" extended AGUT by Froggatt–Nielsen (called here as G_f–theory), the new results of G_{ext}–theory by Nielsen–Takanishi [18] are more encouraging.

We conclude that the G–theory, in general, is successful in describing of the SM experiment.

The gauge group G_{ext} undergoes spontaneous breakdown (at some orders of magnitude below the Planck scale) to the Standard Model Group SMG which is the diagonal subgroup of the non–Abelian sector of the group G_{ext}. As was shown in Ref. [18], 6 different Higgs fields: ω, ρ, W, T, ϕ_{WS}, $\phi_{(B-L)}$ break our FRGG–model to the SM. The field ϕ_{WS} corresponds to the Weinberg–Salam Higgs field of Electroweak theory. Its vacuum expectation value (VEV) is fixed by the Fermi constant: $\langle\phi_{WS}\rangle = 246$ GeV, so that we have only 5 free parameters — five VEVs: $\langle\omega\rangle$, $\langle\rho\rangle$, $\langle W\rangle$, $\langle T\rangle$, $\left\langle\phi_{(B-L)}\right\rangle$ to fit the experiment in the framework of the SM. These five adjustable parameters were used with the aim of finding the best fit to experimental data for all fermion masses and mixing angles in the SM, and also to explain the neutrino oscillation experiments.

Typical fit to the masses and mixing angles for the SM leptons and quarks in the framework of the G_{ext}–AGUT is given in Table 2.

Experimental results on solar neutrino and atmospheric neutrino oscillations from Sudbury Neutrino Observatory (SNO Collaboration) and the Super–Kamiokande Collaboration have been used to extract the following parameters:

$$\Delta m^2_{\odot} = m_2^2 - m_1^2, \quad \Delta m^2_{\rm atm} = m_3^2 - m_2^2,$$

$$\tan^2\theta_{\odot} = \tan^2\theta_{12}, \quad \tan^2\theta_{\rm atm} = \tan^2\theta_{23}, \tag{188}$$

where m_1, m_2, m_3 are the hierarchical left-handed neutrino effective masses for the three families. Also the CHOOZ reactor results were used. It is assumed that the fundamental Yukawa couplings in this model are of order unity and so the authors make order of magnitude predictions. The typical fit is shown in Table 2. As we can see, the 5 parameter order of magnitude fit is very encouraging.

There are also 3 see–saw heavy neutrinos in this model (one right–handed neutrino in each family) with masses: M_1, M_2, M_3. The model predicts the following neutrino masses:

$$m_1 \approx 1.4\times 10^{-3}\ {\rm eV}, \quad m_2 \approx 9.6\times 10^{-3}\ {\rm eV}, \quad m_3 \approx 4.2\times 10^{-2}\ {\rm eV} \tag{189}$$

— for left-handed neutrinos, and

$$M_1 \approx 1.0 \times 10^6 \text{ GeV}, \quad M_2 \approx 6.1 \times 10^9 \text{ GeV}, \quad M_3 \approx 7.8 \times 10^9 \text{ GeV} \tag{190}$$

— for right-handed (heavy) neutrinos.

Finally, we conclude that theory with the FRGG–symmetry is very successful in describing experiment.

The best fit gave the following values for VEVs:

$$\langle W \rangle \approx 0.157, \quad \langle T \rangle \approx 0.077, \quad \langle \omega \rangle \approx 0.244, \quad \langle \rho \rangle \approx 0.265 \tag{191}$$

in the "fundamental units", $M_{Pl} = 1$, and

$$\langle \phi_{B-L} \rangle \approx 5.25 \times 10^{15}\,\text{GeV} \tag{192}$$

which gives the see–saw scale: the scale of breakdown of the $U(1)_{(B-L)}$ groups ($\sim 5\times 10^{15}$ GeV).

10. Evolution of running fine structure constants

Let us consider now the evolution of the SM running fine structure constants. The usual definition of the SM coupling constants:

$$\alpha_1 = \frac{5}{3}\frac{\alpha}{\cos^2\theta_{\overline{MS}}}, \quad \alpha_2 = \frac{\alpha}{\sin^2\theta_{\overline{MS}}}, \quad \alpha_3 \equiv \alpha_s = \frac{g_s^2}{4\pi}, \tag{193}$$

where α and α_s are the electromagnetic and $SU(3)$ fine structure constants, respectively, is given in the Modified minimal subtraction scheme ($\overline{MS}$). Here $\theta_{\overline{MS}}$ is the Weinberg weak angle in $\overline{MS}$ scheme. Using RGE with experimentally established parameters, it is possible to extrapolate the experimental values of three inverse running constants $\alpha_i^{-1}(\mu)$ (here μ is an energy scale and $i = 1,\, 2,\, 3$ correspond to $U(1)$, $SU(2)$ and $SU(3)$ groups of the SM) from the Electroweak scale to the Planck scale. The precision of the LEP data allows to make this extrapolation with small errors (see [89]). Assuming that these RGEs for $\alpha_i^{-1}(\mu)$ contain only the contributions of the SM particles up to $\mu \approx \mu_{Pl}$ and doing the extrapolation with one Higgs doublet under the assumption of a "desert", the following results for the inverses $\alpha^{-1}_{Y,2,3}$ (here $\alpha_Y \equiv (3/5)\alpha_1$) were obtained in Ref. [8] (compare with [89]):

$$\alpha_Y^{-1}\left(\mu_{Pl}\right) \approx 55.5; \quad \alpha_2^{-1}\left(\mu_{Pl}\right) \approx 49.5; \quad \alpha_3^{-1}\left(\mu_{Pl}\right) \approx 54.0. \tag{194}$$

The extrapolation of $\alpha^{-1}_{Y,2,3}(\mu)$ up to the point $\mu = \mu_{Pl}$ is shown in Fig. 20.

10.1. "Gravitational fine structure constant" evolution

In this connection, it is very attractive to include gravity. The quantity:

$$\alpha_g = \left(\frac{\mu}{\mu_{Pl}}\right)^2 \tag{195}$$

plays the role of the running "gravitational fine structure constant" (see Ref. [91]) and the evolution of its inverse is presented in Fig. 21 together with the evolutions of $\alpha_i^{-1}(\mu)$ where $i = 1, 2, 3$.

Then we see the intersection of $\alpha_g^{-1}(\mu)$ with $\alpha_1^{-1}(\mu)$ at the point:

$$\left(x_0, \alpha_0^{-1}\right),$$

where ($x_0 = \log_{10}(\mu_{int.})$):

$$x_0 \approx 18.3, \quad \alpha_0^{-1} \approx 34.4. \tag{196}$$

11. Monopoles in the SM and FRGGM

11.1. Renormalization group equations for electric and magnetic fine structure constants

J. Schwinger was first [92] who investigated the problem of renormalization of the magnetic charge in Quantum Electro-Magneto Dynamics (QEMD), i.e. in the Abelian quantum field theory of electrically and magnetically charged particles (with charges e and g, respectively).

Considering the "bare" charges e_0 and g_0 and renormalised (effective) charges e and g, Schwinger (and later the authors of Refs. [93] and [94]) obtained:

$$\frac{e}{g} = \frac{e_0}{g_0}, \tag{197}$$

what means the absence of the Dirac relation [95] for the renormalised electric and magnetic charges.

But there exists another solution of this problem (see Refs. [96–99] and review [100]), which gives:

$$eg = e_0 g_0 = 2\pi n, \quad n \in Z, \tag{198}$$

i.e. the existence of the Dirac relation (charge quantization condition) for both, bare and renormalised electric and magnetic charges. Here we have $n = 1$ for the minimal (elementary) charges.

These two cases lead to the two possibilities for the renormalization group equations (RGEs) describing the evolution of electric and magnetic fine structure constants (52), which obey the following RGEs containing the electric and magnetic β–functions:

$$\frac{d\left(\log(\alpha(\mu))\right)}{dt} = \pm\frac{d\left(\log\left(\tilde{\alpha}(\mu)\right)\right)}{dt} = \beta^{(e)}(\alpha) \pm \beta^{(m)}\left(\tilde{\alpha}\right). \qquad (199)$$

In Eq. (199) we have:

$$t = \log\left(\frac{\mu^2}{\mu_R^2}\right), \qquad (200)$$

where μ is the energy scale and μ_R is the renormalization point.

The second possibility (with minuses) in Eq. (199) corresponds to the validity of the Dirac relation (198) for the renormalised charges. We believe only in this case considered by authors in Ref. [99] where it was used the Zwanziger formalism of QEMD [101–103]. In the present paper, excluding the Schwinger's renormalization condition (197), we assume only the Dirac relation for running α and $\tilde{\alpha}$: $\alpha\tilde{\alpha} = 1/4$.

It is necessary to comment that RGEs (199) are valid only for $\mu > \mu_{\text{threshold}} = m_{mon}$, where m_{mon} is the monopole mass.

In contrast to the method given in Ref. [99], there exists a simple way [29] to obtain Eq. (199) for single electric and magnetic charges of the same type (scalar or fermionic). The general expressions for RGEs are:

$$\frac{d\left(\log(\alpha(\mu))\right)}{dt} = \beta_1(\alpha) + \beta_2\left(\tilde{\alpha}\right) + C, \qquad (201)$$

$$\frac{d\left(\log\left(\tilde{\alpha}(\mu)\right)\right)}{dt} = \tilde{\beta}_1(\alpha) + \tilde{\beta}_2\left(\tilde{\alpha}\right) + \tilde{C}, \qquad (202)$$

The Dirac relation (53) gives:

$$\frac{d\left(\log(\alpha(\mu))\right)}{dt} = -\frac{d\left(\log\left(\tilde{\alpha}(\mu)\right)\right)}{dt}. \qquad (203)$$

Using Eq. (203) and the duality symmetry of QEMD, i.e. the symmetry under the interchange:

$$\alpha \longleftrightarrow \tilde{\alpha}, \qquad (204)$$

it is not difficult to obtain:

$$C = \tilde{C} = 0, \quad \beta_1(\alpha) = -\beta_2(\alpha) = \tilde{\beta}_1(\alpha) = -\tilde{\beta}_2(\alpha) = \beta(\alpha), \qquad (205)$$

and we have the following RGE:

$$\frac{d\left(\log(\alpha(\mu))\right)}{dt} = -\frac{d\left(\log\left(\tilde{\alpha}(\mu)\right)\right)}{dt} = \beta(\alpha) - \beta\left(\tilde{\alpha}\right). \qquad (206)$$

If monopole charges, together with electric ones, are sufficiently small, then β–functions can be considered by the perturbation theory:

$$\beta(\alpha) = \beta_2\left(\frac{\alpha}{4\pi}\right) + \beta_4\left(\frac{\alpha}{4\pi}\right)^2 + ... \qquad (207)$$

and

$$\beta(\tilde{\alpha}) = \beta_2\left(\frac{\tilde{\alpha}}{4\pi}\right) + \beta_4\left(\frac{\tilde{\alpha}}{4\pi}\right)^2 + ... \tag{208}$$

with (see paper [99] and references there):

$$\beta_2 = \frac{1}{3} \quad \text{and} \quad \beta_4 = 1 \quad - \quad \text{for scalar particles,} \tag{209}$$

and

$$\beta_2 = \frac{4}{3} \quad \text{and} \quad \beta_4 \approx 4 \quad - \quad \text{for fermions.} \tag{210}$$

These cases were investigated in Ref. [99]. For scalar electric and magnetic charges we have [99]:

$$\frac{d\left(\log(\alpha(\mu))\right)}{dt} = -\frac{d\left(\log\left(\tilde{\alpha}(\mu)\right)\right)}{dt} = \beta_2\frac{\alpha - \tilde{\alpha}}{4\pi}\left(1 + 3\frac{\alpha + \tilde{\alpha}}{4\pi} + ...\right) \tag{211}$$

with $\beta_2 = 1/3$, and approximately the same result is valid for fermionic particles with $\beta_2 = 4/3$. Eq. (211) shows that there exists a region when both fine structure constants are perturbative. Approximately this region is given by the following inequalities:

$$0.2 \lesssim (\alpha,\, \tilde{\alpha}) \lesssim 1. \tag{212}$$

Using the Dirac relation (53), we see from Eq. (211) that in the region (212) the two–loop contribution is not larger than 30% of the one–loop contribution, and the perturbation theory can be realized in this case (see Refs. [23–29]).

It is necessary to comment that the region (212) almost coincides with the region of phase transition couplings obtained in the lattice $U(1)$–gauge theory (see Subsection 4.4.).

The Zwanziger–like formalism for non–Abelian theories was considered in Ref. [30].

Here we want to discuss the comment given in Ref. [104] where the authors argue that the Dirac relation is a consequence of the non–perturbative effects and cannot be calculated perturbatively. We insist that the Dirac relation exists always when we have vortices in the phase. In the region of charges (212), obtained in QED, the vortices are created for perturbative values of non–dual and dual charges. We always have the Dirac relation, because we always have the confinement phase: in dual, or non–dual sector, where the different types of vortices exist.

11.2. Diminishing of the monopole charge in the FRGGM

There is an interesting way out of this problem if one wants to have the existence of monopoles, namely to extend the SM gauge group so cleverly that certain selected linear combinations of charges get bigger electric couplings than the corresponding SM couplings. That could make the monopoles which, for these certain linear combinations of charges, couple more weakly and thus have a better chance of being allowed "to exist" [105].

An example of such an extension of the SM that can impose the possibility of allowing the existence of free monopoles is just Family Replicated Gauge Group Model (FRGGM).

FRGGs of type $[SU(N)]^{N_{fam}}$ lead to the lowering of the magnetic charge of the monopole belonging to one family:

$$\tilde{\alpha}_{\text{one family}} = \frac{\tilde{\alpha}}{N_{fam}}. \tag{213}$$

For $N_{fam} = 3$ (for $[SU(2)]^3$ and $[SU(3)]^3$) we have:

$$\tilde{\alpha}^{(2,3)}_{\text{one family}} = \frac{\tilde{\alpha}^{(2,\,3)}}{3}.$$

For the family replicated group $[U(1)]^{N_{fam}}$ we obtain:

$$\tilde{\alpha}_{\text{one family}} = \frac{\tilde{\alpha}}{N^*}, \tag{214}$$

where

$$N^* = \frac{1}{2} N_{fam} \left(N_{fam} + 1 \right).$$

For $N_{fam} = 3$ and $[U(1)]^3$, we have:

$$\tilde{\alpha}^{(1)}_{\text{one family}} = \frac{\tilde{\alpha}^{(1)}}{6}$$

— six times smaller! This result was obtained previously in Ref. [8].

11.3. The FRGGM prediction for the values of electric and magnetic charges

By reasons considered at the end of this review, we prefer not to use the terminology "Anti–grand unification theory, i.e. AGUT", but call the FRGG–theory with the group of symmetry G, or G_f, or G_{ext}, given by Eqs. (183–186) as "G–theory", because as it is shown below, we have a possibility of the Grand Unification near the Planck scale using just this theory.

According to the FRGGM, at some point $\mu = \mu_G < \mu_{Pl}$ (but near μ_{Pl}) the fundamental group G (or G_f, or G_{ext}) undergoes spontaneous breakdown to its diagonal subgroup:

$$G \longrightarrow G_{diag.\,subgr.} = \{g, g, g || g \in SMG\}, \tag{215}$$

which is identified with the usual (low–energy) group SMG. The point $\mu_G \sim 10^{18}$ GeV also is shown in Fig. 20, together with a region of G–theory, where AGUT works.

The AGUT prediction of the values of $\alpha_i(\mu)$ at $\mu = \mu_{Pl}$ is based on the MPM assumption about the existence of the phase transition boundary point MCP at the Planck scale,

and gives these values in terms of the corresponding critical couplings $\alpha_{i,crit}$ [8, 14, 105] (see Eqs. (213, 214)):

$$\alpha_i(\mu_{Pl}) = \frac{\alpha_{i,\,crit}}{N_{gen}} = \frac{\alpha_{i,\,crit}}{3} \quad \text{for} \quad i = 2, 3, \tag{216}$$

and

$$\alpha_1(\mu_{Pl}) = \frac{\alpha_{1,\,crit}}{\frac{1}{2}N_{gen}\,(N_{gen}+1)} = \frac{\alpha_{1,\,crit}}{6} \quad \text{for} \quad U(1). \tag{217}$$

There exists a simple explanation of the relations (216) and (217). As it was mentioned above, the group G breaks down at $\mu = \mu_G$. It should be said that at the very high energies $\mu_G \le \mu \le \mu_{Pl}$ (see Fig. 20) each generation has its own gluons, own W's, etc. The breaking makes only linear combination of a certain color combination of gluons which exists in the SM below $\mu = \mu_G$ and down to the low energies. We can say that the phenomenological gluon is a linear combination (with amplitude $1/\sqrt{3}$ for $N_{gen} = 3$) for each of the AGUT gluons of the same color combination. This means that coupling constant for the phenomenological gluon has a strength that is $\sqrt{3}$ times smaller, if as we effectively assume that three AGUT $SU(3)$ couplings are equal to each other. Then we have the following formula connecting the fine structure constants of G–theory (e.g. AGUT) and low energy surviving diagonal subgroup $G_{diag.\,subg.} \subseteq (SMG)^3$ given by Eq. (215):

$$\alpha^{-1}_{diag.,\,i} = \alpha^{-1}_{1st\,gen.,\,i} + \alpha^{-1}_{2nd\,gen.,\,i} + \alpha^{-1}_{3rd\,gen.,\,i}. \tag{218}$$

Here $i = U(1)$, $SU(2)$, $SU(3)$, and $i = 3$ means that we talk about the gluon couplings. For non–Abelian theories we immediately obtain Eq. (216) from Eq. (218) at the critical point MCP.

In contrast to non–Abelian theories, in which the gauge invariance forbids the mixed (in generations) terms in the Lagrangian of G–theory, the $U(1)$–sector of AGUT contains such mixed terms:

$$\frac{1}{g^2}\sum_{p,\,q} F^2_{\mu\nu,\,pq} = \frac{1}{g^2_{11}}F^2_{\mu\nu,\,11} + \frac{1}{g^2_{12}}F^2_{\mu\nu,\,12} + \ldots + \frac{1}{g^2_{23}}F^2_{\mu\nu,\,23} + \frac{1}{g^2_{33}}F^2_{\mu\nu,\,33}, \tag{219}$$

where $p,\ q = 1,\ 2,\ 3$ are the indices of three generations of the AGUT group $(SMG)^3$. The last equation explains the difference between the expressions (216) and (217).

It was assumed in Ref. [8] that the MCP values $\alpha_{i,\,crit}$ in Eqs. (216) and (217) coincide with the triple point values of the effective fine structure constants given by the generalized lattice $SU(3)$–, $SU(2)$– and $U(1)$–gauge theories described by Eqs. (41) and (42). Also it was used a natural assumption that the effective α_{crit} does not change its value (at least too much) along the whole bordeline "3" of Fig. 10 for the phase transition "Coulomb–confinement" in the $U(1)$ lattice gauge theory with the generalized (two parameters) lattice Wilson action (42).

Now let us consider $\alpha_Y^{-1}(\approx \alpha^{-1})$ at the point $\mu = \mu_G \sim 10^{18}$ GeV shown in Fig. 20. If the point $\mu = \mu_G$ is very close to the Planck scale $\mu = \mu_{Pl}$, then according to Eqs. (194)

and (217), we have:

$$\alpha^{-1}_{1st\,gen.} \approx \alpha^{-1}_{2nd\,gen.} \approx \alpha^{-1}_{3rd\,gen.} \approx \frac{\alpha_Y^{-1}\left(\mu_G\right)}{6} \approx 9, \tag{220}$$

what is almost equal to the value (57):

$$\alpha^{-1}_{crit.,\,theor} \approx 8.5 \tag{221}$$

obtained by the "Parisi improvement method" (see Fig. 14). This means that in the $U(1)$ sector of AGUT we have α_Y (or α_1) near the critical point. Therefore, we can expect the existence of MCP at the Planck scale.

11.4. Evolution of the running fine structure constant in the $U(1)$ theory with monopoles

Considering the evolution of the running $U(1)$ fine structure constant

$$\alpha_Y(\mu) = \frac{\alpha(\mu)}{cos^2\theta_{\overline{MS}}},$$

we have in the SM the following one–loop approximation:

$$\alpha_Y^{-1}(\mu) = \alpha_Y^{-1}\left(\mu_R\right) + \frac{b_Y}{4\pi}t, \tag{222}$$

where t is the evolution variable (5), and b_Y is given for N_{gen} generations and N_S Higgs bosons by the following expression:

$$b_Y = -\frac{20}{9}N_{gen} - \frac{1}{6}N_S. \tag{223}$$

The evolution of $\alpha_Y^{-1}(\mu)$ is represented in Fig. 20 for $N_{gen} = 3$ and $N_S = 1$ by the straight line going up to $\mu = \mu_{Pl}$.

Let us consider now the exotic (not existing in reality) case when we have, for example, the cut–off energy $\mu_{\rm cut-off} \sim 10^{42}$ GeV [29]. In this case the evolution of ${\alpha_Y}^{-1}(\mu)$ is given by Fig. 22, where the straight line 1 (one–loop approximation) goes to the Landau pole at $\alpha_Y^{-1} = 0$. But it is obvious that in the vicinity of the Landau pole, when $\alpha_Y^{-1}(\mu) \to 0$, the charge of $U(1)$ group becomes larger and larger with increasing of μ. This means that the one–loop approximation for β–function is not valid for large μ, and the straight line 1 may change its behaviour. In general, the two–loop approximation for β–function of QED (see Refs. [36, 70, 75] and [106]) shows that this straight line turns and goes down.

Exotically we can consider our space-time as a lattice with parameter a smaller than the Planck scale value λ_P. For example, we can imagine a lattice with $a \sim \left(10^{42}\,{\rm GeV}\right)^{-1}$, or the existence of the fundamental magnetically charged Higgs scalar field in the vicinity

of large $\mu_{crit} \sim 10^{38}$ GeV, when we have the phase transition point with $\alpha^{-1}_{Y,crit} \approx 5$ (see below (230) and Fig. 22).

The artifact monopoles, responsible for the confinement of electric charges at the very small distances, can be approximated by the magnetically charged Higgs scalar field, which leads to the confinement–deconfinement phase transition, as it was shown in Refs. [23–29]. If we have this phase transition, then there exists a rapid fall in the evolution of $\alpha^{-1}(\mu)$ (α^{-1}_Y or $\alpha^{-1}_{Y,G}$, etc.) near the phase transition (critical) point. This "fall" always accompanies the phase transition from the Coulomb–like phase to the confinement one.

Indeed, we can present the effective Lagrangian of our field system as a function of the variable

$$F^2 \equiv F^2_{\mu\nu}, \tag{224}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{225}$$

is the field strength tensor:

$$L_{eff} = -\frac{\alpha^{-1}_{eff}(F^2)}{16\pi} F^2. \tag{226}$$

When $F^2 = \vec{B}^2$ ($\vec{B}$ is the magnetic field) and $\vec{B}^2$ is independent of space–time coordinates (see Refs. [107–109]), we can write the effective potential:

$$V_{eff} = \frac{\alpha^{-1}_{eff}\left(\vec{B}^2\right)}{16\pi} \vec{B}^2 = A\alpha^{-1}_{eff}(t) e^{2t}. \tag{227}$$

Here, choosing $\vec{B}^2 = \mu^4$ and $\mu_R = \mu_{\text{cut-off}}$, we have:

$$A = \frac{\mu^4_{\text{cut-off}}}{16\pi},$$

and

$$t = \frac{1}{2}\log\left(\frac{F^2}{\mu^4_{\text{cut-off}}}\right). \tag{228}$$

It is well–known [107, 108] that $\alpha^{-1}_{eff}(t)$ has the same evolution over $t = \log\left(\mu^2/\mu^2_{\text{cut-off}}\right)$ whether we consider $\mu^2 = p^2$ (where p is the 4-momentum), or $\mu^4 = F^2 = \vec{B}^2$ (at least, up to the second order perturbation).

In the confinement region ($t > t_{crit}$) the effective potential (227) has a minimum, given by the requirement [108]:

$$\left[\frac{d\alpha^{-1}_{eff}}{dt} + 2\alpha^{-1}_{eff}(t)\right]\Bigg|_{t=t_{min}} = 0. \tag{229}$$

Of course, we need this minimum for t–values above t_{crit} in order to have confinement which namely means that we have a nonzero $F^2 = \vec{B}^2_0 = const$ in the vacuum [65–67, 81].

This minimum of the effective potential can exist only if $\alpha_{eff}^{-1}(\mu)$ has a rapid fall near the phase transition point which is illustrated in Figs. 22, 23 by curve 2.

The existence of minimum of the effective potential (227) explains why the straight line 1 changes its behaviour and rapidly falls: $\alpha_{eff}^{-1}(t)$ is multiplied by $\exp(2t)$ in Eq. (227).

After this "fall" $\alpha_Y^{-1}(\mu)$ has a crook and goes to the constant value, as it is shown in Fig. 22 by solid curve 2, demonstrating the phase transition from Coulomb–like phase to the confinement one.

The next step is to give the explanation why $\alpha_{eff}^{-1}(t)$ is arrested when $t \to t_{\rm cut-off}$.

The process of formation of strings in the confinement phase (considered in Ref. [23]) leads to the "freezing" of α: in the confinement phase the effective electric fine structure constant is almost unchanged and approaches its maximal value $\alpha = \alpha_{max}$ when $(\mu \to \infty)$. It was shown in Subsection 4.4. that the authors of Ref. [65] predicted the maximal value: $\alpha_{max} \approx \pi/12 \approx 0.26$, due to the Casimir effect for strings.

Fig. 12 demonstrates the tendency to freezing of α in the compact QED for $\beta < \beta_T$ (i.e. for "bare" constant $e_0 > 1$, what means $\alpha_0^{-1} < 4\pi \approx 12.56$).

Choosing the lattice result (51), which almost coincides with the HMM result (104), we have:

$$\alpha_Y^{-1}\left(\mu_{crit}\right) \equiv \alpha_{U(1),\,crit}^{-1} \approx 5. \tag{230}$$

Analogously using (59) we have the maximal value for α_Y^{-1}:

$$\alpha_{Y,\,max}^{-1} \approx \frac{1}{0.26} \approx 3.8. \tag{231}$$

An interesting situation arises in the theory with FRGG–symmetry, when it begins to work at $\mu = \mu_G\,(< \mu_{Pl})$. As it was shown in Subsection 11.2., in the vicinity of the phase transition point the $U(1)$–sector of FRGG has $\alpha_{Y,\,G} \equiv \alpha_{Y,\,one\,fam.}$, which is 6 times larger than α_Y. Now the phase transition "deconfinement–confinement" occurs at $\mu_{crit} = \mu_{Pl}$ (but not at $\mu_{crit} \sim 10^{38}$ GeV as it was in the SM prolonged up to the scale $\mu_{\rm cut-off} \sim 10^{42}$ GeV). This case is not "exotic" more, and confirms the MPP idea [8].

The evolution of the one family inverse fine structure constant $\alpha_{Y,\,G}(\mu)^{-1}(x)$ is given in Fig. 23, which demonstrates the existence of critical point at the Planck scale.

Here it is necessary to comment that we have given a qualitative behaviour of the fall in Figs. 22, 23 believing in the confinement existing in the $U(1)_Y$ theory. In general, the existence of the confinement, as well as the shape of the fall (wide or narrow), depends on the type of theory considered. If the cut–off energy is not very high ($\mu_{\rm cut-off} < \mu_{crit}$), or the lattice spacing a is not too small, then there is no confinement region in such a theory (for example, in the $U(1)$ sector of the SM we have $\mu_{\rm cut-off} = \mu_{Pl}$ which is smaller than μ_{crit} given by Fig. 22 and the confinement phase is not available).

According to MPP [1], Nature has to have the phase transition point at the Planck scale not only for the Abelian $U(1)$ theory, but also for non–Abelian theories. This means that the effective potential of the $SU(3)$ gauge theory has the second minimum at the Planck scale (the first one corresponds to the low–energy hadron physics). String states of this second confinement phase are not observed in Nature, because the FRGG–theory approaches the confinement phase at the Planck scale, but does not reach it.

11.5. Olive's monopoles

The fact that we have one special monopole for each of the three groups $SU(3)$, $SU(2)$ and $U(1)$, which we have considered calculating the phase transition (critical) couplings (for the confinement due to monopole condensation, either in the SM, or in FRGGM) is indeed not consistent with quarks and leptons as phenomenologically found objects. The point is that the $SU(3)$ monopole, for instance, radiates a flux corresponding to a path in the gauge group $SU(3)$ from the unit element to one of the non–trivial center elements. Such a monopole gives rise to a phase factor $\exp(2\pi i/3)$ when a quark encircles its Dirac string. Therefore, it does not allow quarks.

What is allowed consistently with the SM representations is the object which was proposed by D. Olive [110] and called "the Olive–monopole". These monopoles have the magnetic charge under *all three subgroups:* $SU(3)$, $SU(2)$ *and* $U(1)$ of SMG. Their total magnetic charge corresponds to the center element $\left(Ie^{i2\pi/3},\, -I,\, 2\pi\right) \in SU(3)\times SU(2)\times \mathbf{R}$ contained in the covering group $SU(3) \times SU(2) \times \mathbf{R}$ of SMG, which according to the interpretation of Ref. [111], has a meaning of the *Lie group*, rather than just Lie algebra, fitting the SM representations. That is, Olive–monopole in the SM has at the same time three different magnetic charges with the following sizes:

1. An $SU(3)$ magnetic charge identical to the one that would allow only the representations of the <u>group</u> $SU(3)/Z_3$, i.e. only the representations with triality t=0.

2. An $SU(2)$ magnetic charge identical to the one that would allow only representations of the <u>group</u> $SU(2)/Z_2 = SO(3)$, i.e. only the representations with integer weak isospin.

3. And finally, a $U(1)$ weak hypercharge monopolic charge of a size that if alone would allow only integer values of the weak hypercharge half, i.e. of $y/2 =$ integer.

These three magnetic charge contributions would, if alone, not allow the existence neither fermions, nor the Higgs bosons in the SM. However, considering the phase of a

[1]We call it MPP-II to distinguish this definition from the version in which MPP-I is defined only as a phase transition depending on bare couplings while no scale involved

quark or lepton field along a little circle encircling the Dirac string for the Olive's SM–monopole, one gets typically a phase rotation from each of the three contributions to the magnetic charge. The consistency condition to have the Dirac string without visible effect is that these phase contributions *together* make up a multiple of 2π. It can be checked that the quark and lepton representations, as well as the Weinberg–Salam Higgs boson representation, lead to the full phase rotations which are indeed a multiple of 2π.

Thus, as it was already mentioned above, if we imagine monopoles with each of these contributions alone they would not allow neither the phenomenologically observed quarks and leptons, nor the Higgs bosons.

In going to the FRGGM we can, without problem, postulate one Olive–monopole for each proto–family since the proto–family representations are just analogous to the ones in the SM.

Considering the Olive–monopoles condensation (causing a confinement-deconfinement phase transition) for different families, we assume that as long as we consider, for example, only the $SU(3)$–coupling to cause the phase transition, the Olive–monopole functions as if it is the $SU(3)$–monopole consistent only with the representations of $SU(3)/Z_3$. But this is just what gives the phase transition couplings derived with help of Eqs. (230) and (221). Similarly, it is easy to see that the use of the Olive–monopole for all gauge groups $SU(3)$, $SU(2)$, $U(1)$ leads to the phase transition couplings obtained by combining Eqs. (221) and (230).

12. Anti–GUT prediction of coupling constants near the Planck scale

As it was mentioned above, the lattice investigators were not able to obtain the lattice triple point values of $\alpha_{i,\,crit}$ ($i = 1,\, 2,\, 3$ correspond to $U(1)$, $SU(2)$ and $SU(3)$ groups) by Monte Carlo simulation methods. These values were calculated theoretically by Bennett and Nielsen in Ref. [8]. Using the lattice triple point values of $(\beta_A;\, \beta_f)$ and $\left(\beta^{lat};\, \gamma^{lat}\right)$ (see Figs. 5, 6 and Fig. 10), they have obtained $\alpha_{i,\,crit}$ by the "Parisi improvement method":

$$\alpha^{-1}_{Y,\,crit} \approx 9.2 \pm 1, \quad \alpha^{-1}_{2,\,crit} \approx 16.5 \pm 1, \quad \alpha^{-1}_{3,\,crit} \approx 18.9 \pm 1. \tag{232}$$

Assuming the existence of MCP at $\mu = \mu_{Pl}$ and substituting the last results in Eqs. (216) and (217), we have the following prediction of AGUT [8]:

$$\alpha_Y^{-1}\left(\mu_{Pl}\right) \approx 55 \pm 6; \quad \alpha_2^{-1}\left(\mu_{Pl}\right) \approx 49.5 \pm 3; \quad \alpha_3^{-1}\left(\mu_{Pl}\right) \approx 57.0 \pm 3. \tag{233}$$

These results coincide with the results (194) obtained by the extrapolation of experimental data to the Planck scale in the framework of the pure SM (without any new particles) [8, 89].

Using the relation (179), we obtained the result (180), which in our case gives the following relations [29]:

$$\alpha_{Y,\,crit}^{-1} : \alpha_{2,\,crit}^{-1} : \alpha_{3,\,crit}^{-1} = 1 : \sqrt{3} : \frac{3}{\sqrt{2}} = 1 : 1.73 : 2.12. \tag{234}$$

Let us compare now these relations with the MPM prediction.

For $\alpha_{Y,\,crit}^{-1} \approx 9.2$ given by the first equation of (232), we have:

$$\alpha_{Y,\,crit}^{-1} : \alpha_{2,\,crit}^{-1} : \alpha_{3,\,crit}^{-1} = 9.2 : 15.9 : 19.5. \tag{235}$$

In the framework of errors, the last result coincides with the AGUT–MPM prediction (232). Of course, it is necessary to take into account an approximate description of the confinement dynamics in the $SU(N)$ gauge theories, developed by our investigations.

13. The possibility of the Grand Unification near the Planck scale

In the Anti-grand unified theory (AGUT) [11–21] the FRGG breakdown was considered at $\mu_G \sim 10^{18}$ GeV. It is a significant point for MPM. In this case the evolutions of the fine structure constants $\alpha_i(\mu)$ exclude the existence of the unification point up to the Planck scale (also in the region $\mu > \mu_G$).

The aim of this Section is to show, as in Ref. [29], that we have quite different consequences of the extension of the SM to FRGGM if G–group undergoes the breakdown to its diagonal subgroup (i.e. SM) not at $\mu_G \sim 10^{18}$ GeV, but at $\mu_G \sim 10^{14}$ or 10^{15} GeV, i.e. before the intersection of $\alpha_2^{-1}(\mu)$ with $\alpha_3^{-1}(\mu)$ at $\mu \approx 10^{16}$ GeV.

In fact, here we are going to illustrate the idea that with monopoles we can modify the running of the fine structure constants so much that unification can be arranged without needing SUSY. To avoid confinement of monopoles at the cut–off scale we need FRGG or some replacement for it (see Subsection 11.2.).

If we want to realize the behaviour shown in Fig. 23 for the function $\alpha_{Y,\,FRGG}^{-1}(\mu) = \alpha_{Y,\,G}^{-1}(\mu)$ near the Planck scale, then we need the evolution of all $\alpha_i^{-1}(\mu)$ to turn away from asymptotic freedom. This could be achieved by having more fermions, i.e. generations, appearing above the μ_G–scale.

Now we shall suggest that such appearance of more fermions above μ_G is not so unlikely.

13.1. Guessing more particles in the FRGGM

Once we have learned about the not so extremely simple SM, it is not looking likely that the fundamental theory — the true model of everything — should be so simple as to have only one single type of particle — the "urparticle" — unless this "urparticle" should be a particle that can be in many states internally such as say the superstring. Therefore, we should not necessarily assume that the number of species of particles is minimal anymore — as could have been reasonable in a period of science where one had only electron, proton and perhaps neutron, ignoring the photon exchanges which bind the atom together, so that only three particles really existed there. Since now there are too many particles.

Looking at the problem of guessing the physical laws beyond the SM, we should rather attempt to guess a set of species of particles to exist and the order of magnitudes of the numbers of such species which one should find at the various scales of energy. Indeed, the historical learning about the species of particles in Nature has rather been that physicists have learned about many types of particles not much called for at first: It is only rather few of particle types, which physicists know today that have so great significance in the building of matter or other obviously important applications that one could not almost equally well imagine a world without these particles. They have just been found flavour after flavour experimentally studies often as a surprise, and if for instance the charmed quark was needed for making left handed doublets, that could be considered as a very little detail in the weak interactions which maybe was not needed itself.

These remarks are meant to suggest that if we should make our expectations to be more "realistic" in the sense that we should get less surprised next time when the Nature provides us with new and seemingly not needed particles, we should rather than guessing on the minimal system of particles seek to make some more statistical considerations as to how many particle types we should really expect to find in different ranges of energy or mass.

To even crudely attack this problem of guessing it is important to have in mind the reasons for particles having the mass order of magnitudes. In this connection, a quite crucial feature of the SM "zoo" is that except for the Higgs particle itself, are mass protected particles, which have no mass in the limit when the Higgs field has no vacuum expectation value (VEV). In principle, you would therefore expect that the SM particles have masses of order of the Higgs VEV. Actually they are mostly lighter than that by up to five orders of magnitude.

In the light of this mass protection phenomenon it would be really very strange to assume that there should be no other particles than in the SM if one went up the energy scale and looked for heavier particles, because then what one could ask: Why should there be only mass protected particles ? After all, there are lots of possibilities for making vector coupled Dirac particles, say. It would be a strange accident if Nature should only have

mass protected particles and not a single true Dirac particle being vector coupled to even the weak gauge particles. It is much more natural to think that there are at higher masses lots of different particles, mass protected as well as not mass protected. But because we until now only could "see" the lightest ones among them, we only "saw" the mass protected ones.

In this light the estimate of how many particles are to be found with higher masses should now be a question of estimating how many particles turn out mass protected and getting masses which we can afford to "see" today.

We are already in the present article having the picture that as one goes up in the energy scale μ there will be bigger and bigger gauge group, which will thus be able to mass protect more and more particle types. Each time one passes a breaking scale at which a part of the gauge group breaks down — or thought the way from infrared towards the ultraviolet: each time we get into having a new set of gauge particles — there will be a bunch of fermions which are mass protected to get — modulo small Yukawa couplings — masses just of the order of magnitude of the Higgs scale corresponding to that scale of diminishing of the gauge group.

If we for example think of the scale of breaking of our FRGG down to its diagonal subgroup, then we must expect that there should be some fermions just mass protected to that scale. However, these particles should be vector coupled w.r.t. the SM gauge fields, and only mass protected by the gauge fields of the FRGG, which are not diagonal. We would really like to say that it would be rather strange if indeed there were no such particles just mass protected to that scale.

13.2. Quantitative estimate of number of particles in the FRGGM

We might even make an attempt to perform a quantitative estimate of how many particle species we should expect to appear when we pass the scale $\mu = \mu_G$ going above μ_G. Of course, such an estimate can be expected to be very crude and statistical, but we hope that anyway it would be better than the unjustified guess that there should be nothing, although this guess could be in some sense the simplest one.

Since it is going to be dependent on the detailed way of arguing, we should like to make a couple of such estimates:

1. The first estimate is the guessing that, in analogy with the type of particle combinations which we have had in FRGGM already, we find the particles grouped into families which are just copies of the SM families. If only one of the gauge group families is considered, then two others are represented trivially on that family. We

shall, however, allow that these families can easily be mirror families, in the sense that they have the weak doublets being the right handed particles and actually every gauge quantum number parity are reflected. But, by some principle, only small representations are realized in Nature and we could assume away the higher representations. Now let us call the number of mirror plus ordinary families which are present above the scale μ_G of the breakdown of FRGG to the diagonal subgroup as $N_{fam,\,tot}$, and assume that we have a statistical distribution as if these families or mirror families had been made one by one independently of each other with a probability of 50% for it being a mirror family and 50% for it being an ordinary left handed one. The order of the number of families survived under the scale μ_G should then be equal to the order of the difference between two samples of the Poisson distributed numbers with average $N_{fam,\,tot}/2$. In fact, we might consider respectively the number of mirror families and the number of genuine (left) families as such Poisson distributed numbers. The excess of the one type over the other one is then the number of low energy scales surviving families, and the physicists living today can afford to see them. It is well known that crudely this difference is of the order of $\sqrt{N_{fam,\,tot}}$. But we know that this number of surviving families has already been measured to be 3, and so we expect that $\sqrt{N_{fam,\,tot}} = 3$, what gives $N_{fam,\,tot} \approx 9$. This would mean that there are 6 more families to be found above the diagonal subgroup breaking scale μ_G.

2. As an alternative way of estimating, we can say very crudely that the fermions above the scale μ_G could be mass protected by 3 times as many possible gauge quantum numbers, as far as there are three families of gauge boson systems in our FRGGM. If we use the "small representations assumption", then going from $\mu > \mu_G$ to $\mu < \mu_G$, two of three fermions loose their mass protection, and these two fermions obtain masses of order of the scale μ_G. But this means that 1/3 of all fermions survive to get masses below the scale μ_G and become "observable in practice". Again we got that there should be two times as many particles with masses at the diagonal subgroup breaking scale μ_G than at the EW scale. We got the same result by two different ways. Notice though that in both case we have used a phenomenologically supported assumption that Nature prefers very small representations. In fact, it seems to be true that the SM representations are typically the smallest ones allowed by the charge quantization rule:

$$\frac{Y}{2} + \frac{d}{2} + \frac{t}{3} = 0 \pmod 1, \tag{236}$$

where d and t are duality and triality, respectively.

13.3. The FRGGM prediction of RGEs. The evolution of fine structure constants near the Planck scale

Let us consider now, in contrast to the AGUT having the breakdown of G–group at $\mu_G \sim 10^{18}$ GeV, a new possibility of the FRGG breakdown at $\mu_G \sim 10^{14}$ or 10^{15} GeV (that is, before the intersection of $\alpha_2^{-1}(\mu)$ with $\alpha_3^{-1}(\mu)$, taking place at $\mu \sim 10^{16}$ GeV in SM). This possibility was considered in Ref. [29]. Then in the region $\mu_G < \mu < \mu_{Pl}$ we have three SMG $\times U(1)_{(B-L)}$ groups for three FRGG families as in Refs. [11–21].

In this region we have, according to the statistical estimates made in Subsection 13.2., a lot of fermions, mass protected or not mass protected, belonging to usual families or to mirror ones. We designate the total number of these fermions N_F, maybe different with $N_{fam.\,tot}$.

Also monopoles can be important in the vicinity of the Planck scale: they can give essential contributions to RGEs for $\alpha_i(\mu)$ and change the previously considered evolution of the fine structure constants.

Analogously to Eq. (206), obtained in Ref. [99], we can write the following RGEs for $\alpha_i(\mu)$ containing β–functions for monopoles:

$$\frac{d\left(\log\left(\alpha_i(\mu)\right)\right)}{dt} = \beta(\alpha_i) - \beta^{(m)}\left(\tilde{\alpha}_i\right), \quad i = 1, 2, 3. \tag{237}$$

We can use the one–loop approximation for $\beta(\alpha_i)$ because α_i are small, and the two–loop approximation for dual β–function $\beta^{(m)}\left(\tilde{\alpha}_i\right)$ by reason that $\tilde{\alpha}_i$ are not very small. Finally, taking into account that in the non–Abelian sectors of FRGG we have the Abelian artifact monopoles (see Subsection 7.1.), we obtain the following RGEs:

$$\frac{d\left(\alpha_i^{-1}(\mu)\right)}{dt} = \frac{b_i}{4\pi} + \frac{N_M^{(i)}}{\alpha_i}\beta^{(m)}\left(\tilde{\alpha}_{U(1)}\right), \tag{238}$$

where b_i are given by the following values:

$$\begin{aligned} b_i &= (b_1, b_2, b_3) \\ &= \left(-\frac{4N_F}{3} - \frac{1}{10}N_S,\ \frac{22}{3}N_V - \frac{4N_F}{3} - \frac{1}{6}N_S,\ 11N_V - \frac{4N_F}{3}\right). \end{aligned} \tag{239}$$

The integers N_F, N_S, N_V are respectively the total numbers of fermions, Higgs bosons and vector gauge fields in FRGGM considered in our theory, while the integers $N_M^{(i)}$ describe the amount of contributions of scalar monopoles.

The Abelian monopole β–function in the two–loop approximation is:

$$\beta^{(m)}\left(\tilde{\alpha}_{U(1)}\right) = \frac{\tilde{\alpha}_{U(1)}}{12\pi}\left(1 + 3\frac{\tilde{\alpha}_{U(1)}}{4\pi}\right). \tag{240}$$

Using the Dirac relation (53) we have:

$$\beta^{(m)} = \frac{\alpha_{U(1)}^{-1}}{48\pi}\left(1 + 3\frac{\alpha_{U(1)}^{-1}}{16\pi}\right), \tag{241}$$

and the group dependence relation (179) gives:

$$\beta^{(m)} = \frac{C_i {\alpha_i}^{-1}}{48\pi}\left(1 + 3\frac{C_i {\alpha_i}^{-1}}{16\pi}\right), \tag{242}$$

where

$$C_i = (C_1, C_2, C_3) = \left(\frac{5}{3}, \frac{1}{\sqrt{3}}, \frac{\sqrt{2}}{3}\right). \tag{243}$$

Finally we have the following RGEs:

$$\frac{d\left(\alpha_i^{-1}(\mu)\right)}{dt} = \frac{b_i}{4\pi} + N_M^{(i)}\frac{C_i {\alpha_i}^{-2}}{48\pi}\left(1 + 3\frac{C_i {\alpha_i}^{-1}}{16\pi}\right), \tag{244}$$

where b_i and C_i are given by Eqs. (239) and (243), respectively.

In our FRGG model:

$$N_V = 3, \tag{245}$$

because we have 3 times more gauge fields in comparison with the SM ($N_{fam} = 3$).

As an illustration of the *possibility* of unification in an FRGG–scheme with monopoles we propose some–strictly speaking adjusted–parameter choices within the likely ranges already suggested.

In fact, we take the total number of fermions $N_F = 2N_{fam.\,tot}$ (usual and mirror families), $N_{fam.\,tot} = N_{fam}N_{gen} = 3\times 3 = 9$ (three SMG groups with three generations in each group), we have obtained (see Fig. 24) the evolutions of $\alpha_i^{-1}(\mu)$ near the Planck scale by numerical calculations for $N_F = 18$, $N_S = 6$, $N_M^{(1)} = 6$, $N_M^{(2,\,3)} = 3$ and the following $\alpha_i^{-1}(\mu_{Pl})$:

$$\alpha_1^{-1}(\mu_{Pl}) \approx 13, \quad \alpha_2^{-1}(\mu_{Pl}) \approx 19, \quad \alpha_1^{-1}(\mu_{Pl}) \approx 24, \tag{246}$$

which were considered instead of Eq. (232).

We think that the values $N_M^{(i)}$, which we have used here, are in agreement with Eqs. (216) and (217), and $N_S = 6$ shows the the existence of the six scalar Higgs bosons breaking FRGG to SMG (compare with the similar descriptions in Refs. [15–18]).

Fig. 24 shows the existence of the unification point.

We see that a lot of new fermions in the region $\mu > \mu_G$ and monopoles near the Planck scale change the one–loop approximation behaviour of $\alpha_i^{-1}(\mu)$ in SM. In the vicinity of the Planck scale these evolutions begin to decrease, approaching the phase transition (multiple critical) point at $\mu = \mu_{Pl}$ what means the suppression of the asymptotic freedom in the non–Abelian theories.

Here it is necessary to emphasize that these results do not depend on the fact, whether we have in Nature lattice artifact monopoles, or the fundamental Higgs scalar particles with a magnetic charge (scalar monopoles).

Fig. 25 demonstrates the unification of all gauge interactions including gravity (the intersection of α_g^{-1} with α_i^{-1}) at

$$\alpha_{\rm GUT}^{-1} \approx 27 \quad \text{and} \quad x_{\rm GUT} \approx 18.4. \tag{247}$$

Here we can expect the existence of $[SU(5)]^3$ or $[SO(10)]^3$ unification. Of course, the results obtained in Ref. [29] are preliminary and in future it is desirable to perform the spacious investigation of the unification possibility.

Calculating the GUT–values for one family fine structure constants considered in this paper we have for $i = 5$:

$$\alpha_{GUT,\,one\,fam}^{-1} = \frac{\alpha_{\rm GUT}}{3} \approx 9, \tag{248}$$

what corresponds to the Abelian monopole (for the $SU(5)/Z_5$ lattice artifact) coupling with the total average monopolic fine structure constant $\tilde{\alpha}_{eff}$ and the "genuine" monopole fine structure constant $\tilde{\alpha}_{\rm genuine}$, as defined in Ref. [29], determined from Eq. (179) and the Dirac relation by

$$\alpha_N^{-1} = \frac{N}{2}\sqrt{\frac{N+1}{N-1}} \cdot 4\tilde{\alpha}_{eff} = \frac{2N}{N-1}\tilde{\alpha}_{\rm genuine} \tag{249}$$

leading to

$$\tilde{\alpha}_{eff} = 9 \cdot \frac{2}{5 \cdot 4}\sqrt{\frac{4}{6}} = 0.7, \tag{250}$$

and

$$\tilde{\alpha}_{\rm genuine} \approx \frac{2 \cdot 4}{4 \cdot 5} \cdot 9 \approx 3.6. \tag{251}$$

This value $\tilde{\alpha}_{eff}$ suggests that we may apply crude perturbation theory both for monopoles and charges if accepting that in the region (212).

Critical coupling corresponds (see Section 5.) to $\tilde{\alpha}_{eff,\,crit} = 1.20$ giving for $SU(5)/Z_5$

$$\alpha_{5,\,crit}^{-1} \approx \frac{5}{2}\sqrt{\frac{6}{4}} \cdot 4 \cdot 1.20 \approx 14.5, \tag{252}$$

meaning that the unified couplings for $[SU(5)]^3$ are already of confinement strength.

Within the uncertainties we might, however, consider from (248) the coupling strength $\alpha_{\rm GUT} \approx 1/9$ as being equal to the critical value $\alpha_{5,\,crit} \approx 1/14$ from Eq. (252).

If indeed the $\alpha_{\rm GUT}$ were so strong as to suggest confinement at the unification point, it will cause the problem that the fundamental fermions in $SU(5)$ representations would be confined and never show up at lower energy scales.

Assuming the appearance of SUSY, we can expect to see sparticles at the GUT–scale with masses:

$$M \approx 10^{18.4}\,\text{GeV}. \tag{253}$$

Then the scale $\mu_{\rm GUT} = M$ given by Eq. (253) can be considered as a SUSY breaking scale.

The unification theory with $[SU(5)]^3$–symmetry was suggested first by S. Rajpoot [112] (see also [113]).

Note that since our possibility of unification has the very high scale (253), it allows for much longer proton lifetime than corresponding models with more usual unification scales, around 10^{16} GeV. This is true not only for proton decay caused by the gauge boson exchange, but also by the triplet Higgs exchange, since then the mass of the latter also may be put up in the scale.

Considering the predictions of such a theory for the low–energy physics and cosmology, maybe in future we shall be able to answer the question: "Does the unification of $[SU(5)]^3$ or $[SO(10)]^3$ type (SUSY or not SUSY) really exist near the Planck scale ?"

Recently F.S. Ling and P. Ramond [114] considered the group of symmetry $[SO(10)]^3$ and showed that it explains the observed hierarchies of fermion masses and mixings.

14. Discussion of some various scenarios of working MPP

In the present article, except for Section 12., we have taken the picture that the inverse fine structure constants α_i^{-1} run very fast being smaller as μ gets bigger in the interval $[\mu_G,\, \mu_{Pl}]$. The old literature [8] (see Section 12.) considers that the running of the fine structure constants between μ_G and μ_{Pl} is minute. It could be taken, for instance, by the philosophy of Ref. [8], as lowest order of the perturbation theory gives it because the scale ratio logarithm $\log(\mu_G/\mu_{Pl})$ is supposed to be so small that the details of β–functions are hardly of any importance. This point of view is in disagreement with the expectation of quick strong jump put forward in Section 12.. The argument for there having to be such a jump in α_i^{-1} just before $t = 0$ (when $\mu = \mu_{Pl}$) is based on *the assumption that there is a phase transition at the Planck scale as a function of* t. In principle, however, the occurrence of the jump depends on hard computations and on (precisely) *how big the running* $\alpha_i^{-1}(t)$ *are when approaching the Planck scale.* Also that depends on what (matter) particles of Nature exist and influence the β–function (monopoles, extra fermions above μ_G, etc.).

We would like here to list some options for obtaining MCP–agreement in one version or the other one combined with pictures of associated matter:

1. The option of Ref. [8] is to use

    a. the MCP–I definition;

    b. the approximation of very little running between μ_G and μ_{Pl}.

In this interpretation MCP–I, we do not really think of phases as a function of the scale, but only as a function of the bare parameters, e.g. the bare fine structure constants. Rather we have to estimate what are the bare (or the Planck scale) couplings at the phase transition point conceived of as a transition point in the coupling constant space, but not as a function of scale. Here we may simply claim that the Parisi improvement approximation (55) is close to the calculation of the "bare" coupling. The Parisi improvement namely calculates an effective coupling as it would be measured by making small tests of the effective action of the theory on a very local basis just around one plaquette. Indeed, we expect that if in the lattice model one seeks to measure the running coupling at a scale only tinily under the lattice scale, one should really get the result to be the Parisi improved value corrected by a tiny running only.

Since even in this option 1), corresponding to Ref. [8], we want a phase transition which we here like to interpret as due to monopole condensation — in an other phase though — we need to have monopoles. Thus, we get the problem of avoiding these monopoles in the β–function except for an extremely small amount of scales. In the phase in which we live it is suggested that the monopoles are made unimportant in the β–function below μ_G by being *confined* by the Higgs field VEVs which break the FRGG down to the SMG (possibly extended with $U(1)_{(B-L)}$). Since the monopoles, we need, are monopoles for the separate family-gauge-groups and since most of the latter are Higgsed at the μ_G–scale, we expect the monopoles to be confined into hadron–like combinations/bound states by the Higgs fields at the scale μ_G. Thus if μ_G is close to μ_{Pl} — in logarithm — there will be very little contribution of the β–function running due to monopoles.

In such a picture even extra fermions, as suggested in Subsections 13.1. and 13.2., and the SM particles will not give much running between μ_G and μ_{Pl}.

Now there seems to be a discrepancy with calculations in the MCP-II approach (Refs. [23–29] and [54]) which gave the phase transition point what in (57) is called $\alpha^{-1}_{crit.\,lat} \approx 5$. But now we must remember that this value was calculated as a long distance value, i.e. not a "bare" value of the fine structure constant in the lattice calculations of Ref. [54]. Also our Coleman–Weinberg type calculation of it [23–29] was rather a calculation of the renormalised (or dressed) coupling than of a bare coupling.

The disagreement between $\alpha^{-1}_{crit.\,lat} \approx 5$ (= the critical coupling) and the bare coupling in the picture 1 sketched here is suggested to be due to the renormgroup running in other phases caused by monopoles there.

In fact, we have in this picture 1 other phases — existing somewhere else or at some other time — in which there is no breaking down to the diagonal subgroup, as in our phase. In such phases the monopoles for the family groups can be active in the

renormalization group over a longer range of scales provided they are sufficiently light. Assuming that the Schwinger's renormalization scheme is wrong and that the running due to monopoles make the coupling weaker at the higher scales than at the lower energies, it could be *in the other phase* a value corresponding to $\alpha^{-1}_{crit.\,lat} \approx 5$ in Eq. (58) for some lower μ, say at the mass of monopoles, while it is still the Parisi improved value at the bare or fundamental (Planck) scale.

Preliminary calculations indicate that requiring a large positive value of the running $\lambda(\mu)$ at the cut–off scale, i.e. a large positive bare λ_0 in a coupling constant combination at the phase transition makes the value $\alpha_{crit.\,lat} \approx 0.2$ (or $\alpha_{crit} \approx 0.208$ given by Refs. [24–26]) run to a bare α rather close to the Parisi improvement phase transition coupling value (55), giving $\alpha^{-1}_{crit} \approx 8.5$.

In order to explain this picture we need to talk about at least three phases, namely, two phases without the Higgs fields performing the breakdown of the group G to the diagonal subgroup and our own phase. In one of these phases the monopoles condense and provide the "electric" confinement, while the other one has essentially massless gauge particles in the family gauge group discussed. In reality, we need a lot of phases in addition to our own because we need the different combinations of the monopoles being condensed or not for the different family groups.

2. The second picture uses MCP–II, which interprets the phase transition required by any MCP–version as being a phase transition as a function of the scale parameter, μ or t, and the requirement of MCP–II is that it occurs just at the fundamental scale, identified with the Planck scale.

   Since the gauge couplings, if their running is provided (perturbatively) only by the SM particles, would need $\alpha^{-1}_{crit} \approx 9$ rather than $\alpha^{-1}_{crit} \approx 5$, further β–function effects are needed.

   Once the couplings get — as function of t — sufficiently strong, then, of course, perturbation theory gets unjustified, however, and higher orders or non–perturbative effects are important. Indeed, higher order seems to help strengthening of the electric couplings approaching the Planck scale. But most crucially it is argued that the very fact of finding a phase transition at μ_{Pl}, as postulated by MCP-II in itself (see Section 13), suggests that there must be a rather quick running just below μ_{Pl}. This picture 2 has such a property because of the extra fermions — or whatever — which bring the strength of the fine structure constants up for $\mu > \mu_G$, so that non=perturbative and higher order effects can take over and manage to realize MCP-II.

15. Conclusions

In the present review we have developed an idea of the Multiple Point Principle (MPP), according to which several vacuum states with the same energy density exist in Nature. Here the MPP is implemented to the Standard Model (SM), Family replicated gauge group model (FRGGM) and phase transitions in gauge theories (with and without monopoles).

We have shown that the existence of monopoles in Nature leads to the consideration of the FRGGM as an extension of the SM, in the sense that the use of monopoles corresponding to the family replicated gauge fields can bring the monopole charge down from the unbelievably large value which it gets in the simple SM.

In this review:

1. The MPP was put forward as a fine-tuning mechanism predicting the ratio between the fundamental and electroweak (EW) scales in the SM. It was shown that this ratio is exponentially huge:
$$\frac{\mu_{fund}}{\mu_{EW}} \sim e^{40}.$$
2. Using renormalization group equations (RGEs) for the SM, we obtained the effective potential in the two–loop approximation and investigated the existence of its postulated second minimum at the fundamental scale.
3. Lattice gauge theories and phase transitions on the lattice are reviewed.
4. The Dual Abelian Higgs Model of scalar monopoles (Higgs Monopole Model — HMM) is considered as a simplest effective dynamics reproducing a confinement mechanism in the pure gauge lattice theories. It is developed a theory where lattice artifact monopoles are approximated as fundamental point-like particles and described by the Higgs scalar fields.
5. Following the Coleman–Weinberg idea, the RG improvement of the effective potential is used in the HMM with β–functions calculated in the two–loop approximation.
6. The phase transition between the Coulomb–like and confinement phases has been investigated in the $U(1)$ gauge theory. Critical coupling constants were calculated: it was shown that $\alpha_{crit} \approx 0.17$ and $\tilde{\alpha}_{crit} \approx 1.48$ — in the one–loop approximation for β–functions, and $\alpha_{crit} \approx 0.208$ and $\tilde{\alpha}_{crit} \approx 1.20$ — in the two–loop approximation, in agreement with the lattice result:
$$\alpha_{crit}^{lat} = 0.20 \pm 0.015 \quad \text{and} \quad \tilde{\alpha}_{crit}^{lat} = 1.25 \pm 0.10 \quad \text{at} \quad \beta_T \equiv \beta_{crit} \approx 1.011.$$
7. The most significant conclusion for the Multiple Point Model (MPM) is possibly the validity of an approximate universality of the critical couplings. It was shown

that one can crudely calculate the phase transition couplings without using any specific lattice. The details of the lattice, also the details of the regularization, do not matter for values of the phase transition couplings so much. Critical couplings depend only on groups with any regularization. Such an approximate universality is absolutely needed if we want to compare the lattice phase transition couplings with the experimental couplings observed in Nature.

8. The 't Hooft idea about the Abelian dominance in the monopole vacuum of non–Abelian theories is discussed: monopoles of the Yang–Mills theories are the solutions of the $U(1)$–subgroups, arbitrary embedded into the $SU(N)$ group, and belong to the Cartain algebra: $U(1)^{N-1} \in SU(N)$.

9. Choosing the Abelian gauge and taking into account that the direction in the Lie algebra of monopole fields are gauge dependent, it is found an average over these directions and obtained the group dependence relation between the phase transition fine structure constants for the groups $U(1)$ and $SU(N)/Z_N$:
$$\alpha_{N,\,crit}^{-1} = \frac{N}{2}\sqrt{\frac{N+1}{N-1}}\alpha_{U(1),\,crit}^{-1}.$$

10. The Family replicated gauge group model (FRGGM) is reviewed. It is shown that monopoles have N_{fam}, or N^*, times smaller magnetic charge for the gauge group $SU(N)$, or $U(1)$, in the FRGGM than in the SM. Here N_{fam} is a number of families and $N^* = \frac{1}{2}N_{fam}\,(N_{fam}+1)$.

11. Investigating the phase transition in the dual Higgs monopole model, we have pursued two objects: the first aim was to explain the lattice results, but the second one was to confirm the MPM prediction, according to which at the Planck scale there exists a Multiple Critical Point (MCP).

12. The Anti–grand unification theory (AGUT) is reviewed. It is considered that the breakdown of the FRGG at $\mu_G \sim 10^{18}$ GeV leads to the Anti–GUT with the absence of any unification up to the scale $\mu \sim 10^{18}$ GeV.

13. Using the group dependence of critical couplings, we have obtained the following relations:
$$\alpha_{Y,\,crit}^{-1} : \alpha_{2,\,crit}^{-1} : \alpha_{3,\,crit}^{-1} = 1 : \sqrt{3} : \frac{3}{\sqrt{2}} = 1 : 1.73 : 2.12.$$
For $\alpha_{Y,\,crit}^{-1} \approx 9.2$ the last equations give the following result:
$$\alpha_{Y,\,crit}^{-1} : \alpha_{2,\,crit}^{-1} : \alpha_{3,\,crit}^{-1} = 9.2 : 15.9 : 19.5,$$
what confirms the Bennett–Froggatt–Nielsen AGUT–MPM prediction for the SM fine structure constants at the Planck scale:
$$\alpha_Y^{-1}\,(\mu_{Pl}) \approx 55 \pm 6; \quad \alpha_2^{-1}\,(\mu_{Pl}) \approx 49.5 \pm 3; \quad \alpha_3^{-1}\,(\mu_{Pl}) \approx 57.0 \pm 3.$$

14. We have considered the gravitational interaction between two particles of equal masses M, given by the Newtonian potential, and presented the evolution of the quantity:
$$\alpha_g = \left(\frac{\mu}{\mu_{Pl}}\right)^2$$
as a "gravitational fine structure constant".

15. We have shown that the intersection of $\alpha_g^{-1}(\mu)$ with $\alpha_1^{-1}(\mu)$ occurs at the point $\left(x_0, \alpha_0^{-1}\right)$ with the following values:
$$\alpha_0^{-1} \approx 34.4, \quad x_0 \approx 18.3,$$
where $x = \log_{10}(\mu)$ GeV.

16. It is discussed two scenarios implementing Multiple Critical Point (MCP) existence.

17. We have considered the case when our (3+1)–dimensional space–time is discrete and has a lattice–like structure. As a consequence of such an assumption, we have seen that the lattice artifact monopoles play an essential role in the FRGGM near the Planck scale: then these FRGG–monopoles give perturbative contributions to the β–functions of RGEs written for both, electric and magnetic fine structure constants, and change the evolution of $\alpha_i^{-1}(\mu)$ in the vicinity of the Planck scale.

18. Finally, we have investigated the case when the breakdown of FRGG undergoes at $\mu_G \sim 10^{14}, 10^{15}$ GeV and is accompanied by a lot of extra fermions in the region $\mu_G < \mu < \mu_{Pl}$. These extra fermions suppressing the asymptotic freedom of non–Abelian theories lead, together with monopoles, to the possible existence of unification of all interactions including gravity at $\mu_{\rm GUT} = 10^{18.4}$ GeV and $\alpha_{\rm GUT}^{-1} = 27$.

19. It is discussed the possibility of the existence of the family replicated unifications $[SU(5)]^3$ SUSY or $[SO(10)]^3$ SUSY.

In this review we have considered a special case of the new type of unification suggested in Ref. [29]. The realistic family replicated unification theory needs a serious program of investigations giving the predictions for the low–energy physics and cosmology what may be developed in future.

ACKNOWLEDGEMENTS:

We would like to express special thanks to Prof. H.B. Nielsen for the fruitful collaboration and interesting discussions.

One of the authors (L.V.L.) thanks the Institute of Mathematical Sciences of Chennai (India) and personally Prof. N.D. Hari Dass for hospitality and financial support. It is a pleasure to thank all participants of the seminars of this Institute for useful discussions and interest.

We are deeply thankful to D.L. Bennett, C.D. Froggatt, N.D. Hari Dass, R.B. Nevzorov and Y. Takanishi for help, discussions and comments.

This work was supported by the Russian Foundation for Basic Research (RFBR), project N^o 05-02-17642.

References

[1] C.D. Froggatt, H.B. Nielsen, Phys.Lett. **B368**, 96 (1996).

[2] C.D. Froggatt, L.V. Laperashvili, H.B. Nielsen, to appear in Yad.Fiz. **68**, N^o 8 (2005); ArXiv: hep-ph/0407102.

[3] C.D. Froggatt, H.B. Nielsen, *Hierarchy problem and a new bound state*, in: *Proc. to the Euroconference on Symmetries Beyond the Standard Model*, Slovenia, Portoroz, 2003 (DMFA, Zaloznistvo, Ljubljana, 2003), p.73; ArXiv: hep-ph/0312218.

[4] C.D. Froggatt, H.B. Nielsen, L.V. Laperashvili, *Hierarchy problem and a bound state of 6 t and 6 $\bar{t}$*. Invited talk by H.B. Nielsen at the *Coral Gables Conference on Launching of Belle Epoque in High–Energy Physics and Cosmology (CG2003)*, Ft.Lauderdale, Florida, USA, 17–21 Dec., 2003 (see Proceedings); ArXiv: hep-ph/0406110.

[5] C.D. Froggatt, L.V. Laperashvili, H.B. Nielsen, *A new bound state 6t + 6 anti–t and the fundamental–weak scale hierarchy in the Standard Model*, in: *Proceeding to 13th International Seminar on High–Energy Physics 'Quarks 2004'*, Pushkinskie Gory, Russia, 24–30 May, 2004 (World Sci., 2004); ArXiv: hep-ph/0410243.

[6] A.G. Riess *et al.*, Astron.J. **116**, 1009 (1998); S. Perlmutter *et al.*, Astrophys.J. **517**, 565 (1999); C. Bennett *et al.*, ArXiv: astro–ph/0302207; D. Spergel *et al.*, ArXiv: astro–ph/0302209.

[7] D.L. Bennett, C.D. Froggatt, H.B. Nielsen, in: *Proceedings of the 27th International Conference on High Energy Physics*, Glasgow, Scotland, 1994, Ed. by P. Bussey, I. Knowles (IOP Publishing Ltd, 1995), p.557; *Perspectives in Particle Physics '94*, Ed. by D. Klabučar, I. Picek, D. Tadić (World Scientific, Singapore, 1995), p.255; ArXiv: hep-ph/9504294;
C.D. Froggatt, H.B. Nielsen, *Influence from the Future*, ArXiv: hep-ph/9607375.

[8] D.L. Bennett, H.B. Nielsen, Int.J.Mod.Phys. **A9**, 5155 (1994).

[9] D.L. Bennett, H.B. Nielsen, *The Multiple Point Principle: realized vacuum in Nature is maximally degenerate*, in: *Proceedings to the Euroconference on Symmetries Beyond the Standard Model*, Slovenia, Portoroz, 2003 (DMFA, Zaloznistvo, Ljubljana, 2003), p.235.

[10] C.D. Froggatt, H.B. Nielsen, *Trying to understand the Standard Model parameters.* Invited talk by H.B. Nielsen at *the XXXI ITEP Winter School of Physics*, Moscow, Russia, 18–26 Feb., 2003; published in Surveys High Energy Phys. **18**, 55 (2003); ArXiv: hep-ph/0308144.

[11] D.L. Bennett, H.B. Nielsen, I. Picek, Phys.Lett. **B208**, 275 (1988).

[12] C.D. Froggatt, H.B. Nielsen, *Origin of Symmetries*, (World Sci., Singapore, 1991).

[13] L.V. Laperashvili, Yad.Fiz. **57**, 501 (1994) [Phys.At.Nucl. **57**, 471 (1994)]; Yad.Fiz. **59**, 172 (1996) [Phys.Atom.Nucl. **59**, 162 (1996)].

[14] C.D. Froggatt, L.V. Laperashvili, H.B. Nielsen, Y. Takanishi, *Family Replicated Gauge Group Models*, in: *Proceedings of the Fifth International Conference 'Symmetry in Nonlinear Mathematical Physics'*, Kiev, Ukraine, 23–29 June, 2003, Ed. by A.G. Nikitin, V.M. Boyko, R.O. Popovich, I.A. Yehorchenko (Institute of Mathematics of NAS of Ukraine, Kiev, 2004), V.50, Part 2, p.737; ArXiv: hep-ph/0309129.

[15] C.D. Froggatt, G. Lowe, H.B. Nielsen, Phys.Lett. **B311**, 163 (1993); Nucl.Phys. **B414**, 579 (1994); ibid **B420**, 3 (1994); C.D. Froggatt, H.B. Nielsen, D.J. Smith, Phys.Lett. **B235**, 150 (1996); C.D. Froggatt, M. Gibson, H.B. Nielsen, D.J. Smith, Int.J.Mod.Phys. **A13**, 5037 (1998).

[16] C.D. Froggatt, L.V. Laperashvili, H.B. Nielsen, *SUSY or NOT SUSY*, *'SUSY98'*, Oxford, 10–17 July, 1998; hepnts1.rl.ac.uk/susy98/.

[17] H.B. Nielsen, Y. Takanishi, Nucl.Phys. **B588**, 281 (2000); ibid, **B604**, 405 (2001); Phys.Lett. **B507**, 241 (2001); C.D. Froggatt, H.B. Nielsen, Y. Takanishi, Nucl.Phys. **B631**, 285 (2002).

[18] H.B. Nielsen, Y. Takanishi, Phys.Lett. **B543**, 249 (2002).

[19] L.V. Laperashvili, H.B. Nielsen, *Anti–grand unification and critical coupling universality.* A talk given by L.V.L. at *the 8th Lomonosov Conference on Elementary Particle Physics*, Moscow, Russia, 25–29 Aug., 1997; ArXiv: hep-ph/9711388.

[20] L.V. Laperashvili, H.B. Nielsen, Mod.Phys.Lett. **A12**, 73 (1997).

[21] L.V. Laperashvili, *Anti–grand unification and the phase transitions at the Planck scale in gauge theories*, in: *Proceedings of the 4th International Symposium on Frontiers of Fundamental Physics*, Hyderabad, India, 11–13 Dec., 2000, Ed. by B.G. Sidharth (World Scientific, Singapore, 2001); ArXiv: hep-th/0101230.

[22] L.V. Laperashvili, H.B. Nielsen, *Multiple Point Principle and phase transition in gauge theories*, in: *Proceedings of the International Workshop on 'What Comes Beyond the Standard Model'*, Bled, Slovenia, 29 June – 9 July, 1998 (DMFA, Zaloznistvo, Ljubljana, 1999), p.15; ArXiv: hep-ph/9905357.

[23] L.V. Laperashvili, H.B. Nielsen, Int.J.Mod.Phys. **A16**, 2365 (2001); ArXiv: hep-th/0010260.

[24] L.V. Laperashvili, H.B. Nielsen, D.A. Ryzhikh, Int.J.Mod.Phys. **A16**, 3989 (2001); ArXiv: hep-th/0105275.

[25] L.V. Laperashvili, H.B. Nielsen, D.A. Ryzhikh, Yad.Fiz. **65**, 377 (2002) [Phys.At.Nucl. **65**, 353 (2002)]; ArXiv: hep-th/0109023.

[26] L.V. Laperashvili, D.A. Ryzhikh, H.B. Nielsen, *Multiple Point Model and phase transition couplings in the two–loop approximation of dual scalar electrodynamics*,in: *Proceedings 'Bled 2000–2002, What comes beyond the standard model'* (DMFA, Zaloznistvo, Ljubljana, 2002), Vol. 2, pp.131–141; ArXiv: hep-ph/0112183.

[27] L.V. Laperashvili, D.A. Ryzhikh, *Phase transition in gauge theories and the Planck scale physics*, preprint ITEP–24–01, Oct., 2001, 82pp; ArXiv: hep-ph/0212221.

[28] L.V. Laperashvili, D.A. Ryzhikh, $[SU(5)]^3$ *SUSY unification*, hep-preprint dedicated to the 60th jubilee of Holger Bech Nielsen; ArXiv: hep-th/0112142.

[29] L.V. Laperashvili, H.B. Nielsen, D.A. Ryzhikh, DESY–02–188, Nov., 2002. 49pp; Int.J.Mod.Phys. **A18**, 4403 (2003); ArXiv: hep-th/0211224.

[30] L.V. Laperashvili, *Generalized dual symmetry of non–Abelian theories, monopoles and dyons*, in preparation, 2005; see also ArXiv: hep-th/0211227.

[31] C.D. Froggatt, L.V. Laperashvili, R.B. Nevzorov, H.B. Nielsen, ICTP Internal Report IC/IR/2003/16, Italy, Miramare–Trieste, Oct., 2003; Yad.Fiz. **67**, 601 (2004) [Phys.At.Nucl. **67**, 582 (2004)]; ArXiv: hep-ph/0310127.

[32] C.D. Froggatt, L.V. Laperashvili, R.B. Nevzorov, H.B. Nielsen, *No–scale supergravity and the multiple point principle*, in: *Proceedings to the 7th Workshop 'What comes beyond the standard model'*, Bled, Slovenia, 19–31 July, 2004 (DMFA, Zaloznistvo, Ljubljana, M. Breskvar et al., Dec., 2004), pp.17–27; ArXiv: hep-ph/0411273.

[33] C.D. Froggatt, L.V. Laperashvili, R.B. Nevzorov, H.B. Nielsen, M. Sher, *Two Higgs doublet model and Multiple Point Principle*, GUTPA–04–12–01, Dec., 2004, 14pp.; in: *Proceedings to the 7th Workshop 'What Comes Beyond the Standard Model'*, Bled, Slovenia, 19–31 July, 2004 (DMFA, Zaloznistvo, Ljubljana, M. Breskvar et al., Dec 2004), pp.28–39; ArXiv: hep-ph/0412333; ArXiv: hep-ph/0412333; ArXiv: hep-ph/0412208.

[34] L.V. Laperashvili, *The Multiple Point Principle and Higgs bosons*, in: *Proceedings of the International Bogolyubov Conference on Problems of Theoretical and Mathematical Physics*, Moscow–Dubna, Russia, 2–6 Sep., 2004; ArXiv: hep-ph/0411177.

[35] C.R. Das, C.D. Froggatt, L.V. Laperashvili, H.B. Nielsen, *Degenerate vacua, see–saw scale and flipped $SU(5)$*, to be published in 2005.

[36] S. Coleman, E. Weinberg, Phys. Rev. **D7**, 1888 (1973).

[37] M. Sher, Phys.Rep. **179**, 274 (1989).

[38] C.G. Callan, Phys.Rev. **D2**, 1541 (1970).

[39] K. Symanzik, in: *Fundamental Interactions at High Energies*, Ed. A. Perlmutter (Gordon and Breach, New York, 1970).

[40] C.D. Froggatt, H.B. Nielsen, Y. Takanishi, Phys.Rev. **D64**, 113014 (2001).

[41] Particle Data Group, K. Hagiwara et al., Phys.Rev. **D66**, 010001 (2002).

[42] Yu.M. Makeenko, M.I. Polikarpov, *X School in Physics at ITEP*, 1983, No.3, pp. 3, 51;
Yu.M. Makeenko, Uspehi in Physics, **27**, 401 (1984).

[43] K. Wilson, Phys.Rev. **D10**, 2445 (1974).

[44] M. Creutz, I. Jacobs, C. Rebbi, Phys.Rev. **D20**, 1915 (1979).

[45] B. Lautrup, M. Nauenberg, Phys.Lett. **B95**, 63 (1980).

[46] M. Creutz, Phys.Rev. **D21**, 2308 (1980); Phys.Rev.Lett. **45**, 313 (1980).

[47] B. Lautrup, M. Nauenberg, Phys.Rev.Lett. **45**, 1755 (1980).

[48] M. Creutz, Phys.Rev.Lett. **46**, 1441 (1981).

[49] G. Bhanot, M. Creutz, Phys.Rev. **D24**, 3212 (1981).

[50] G. Bhanot, Phys.Lett. B**108**, 337 (1982).

[51] C.P. Bachas, R.F. Dashen, Nucl.Phys. **B210**, 583 (1982).

[52] A.A. Migdal, *Problems and perspectives of gauge theories*, in: E. Seiler,*The phase structure of finite temperature lattice gauge theories*, Munich, Max Planck Institute, 1986 (translated in Russian language).

[53] G. Bhanot, Nucl.Phys. **B205**, 168 (1982); Phys.Rev. **D24**, 461 (1981); Nucl.Phys. **B378**, 633 (1992).

[54] J. Jersak, T. Neuhaus, P.M. Zerwas, Phys.Lett. **B133**, 103 (1983); Nucl.Phys. **B251**, 299 (1985).

[55] N. Arkani–Hamed, Invited talk at the *Conference on Hierarchy Problems in Four and More Dimensions*, ICTP, Italy, Trieste, 1–4 Oct., 2003.

[56] G. Volovik, JETP Lett. **79**, 101 (2004).

[57] G.S. Bali, *Overview from Lattice QCD*, plenary talk presented at *Nuclear and Particle physics with CEBAF at Jefferson Lab*, Dubrovnik, 3–10 Nov., 1998; ArXiv: hep-lat/9901023.

[58] T. Suzuki, Nucl.Phys.Proc.Suppl. **30**, 176 (1993);
R.W.Haymaker, Phys.Rep. **315**, 153 (1999).

[59] M.N. Chernodub, M.I. Polikarpov, in *Confinement, Duality and Non–perturbative Aspects of QCD*, p.387, Ed. by Pierre van Baal, Plenum Press, 1998; ArXiv: hep-th/9710205; M.N. Chernodub, F.V. Gubarev, M.I. Polikarpov, A.I. Veselov, Prog.Ther.Phys.Suppl. **131**, 309 (1998); ArXiv: hep-lat/9802036; M.N. Chernodub, F.V. Gubarev, M.I. Polikarpov, V.I. Zakharov, *Magnetic monopoles, alive*, ArXiv: hep-th/0007135; *Towards Abelian–like formulation of the dual gluodynamics*, ArXiv: hep-th/0010265.

[60] T. Banks, R. Myerson, J. Kogut. Nucl.Phys. **B129**, 493 (1977).

[61] J. Villain. J.de Phys. **36**, 581 (1975).

[62] J.L. Cardy. Nucl.Phys. **B170**, 369 (1980).

[63] J.M. Luck. Nucl.Phys. **B210**, 111 (1982).

[64] G. Parisi, R. Petronzio, F. Rapuano. Phys.Lett. **B128**, 418 (1983);
E. Marinari, M. Guagnelli, M.P. Lombardo, G. Parisi, G. Salina, Proceedings *Lattice '91'*, Tsukuba 1991, p.278–280; Nucl.Phys.Proc.Suppl. **B26**, 278 (1992).

[65] M. Lüscher, K. Symanzik, P. Weisz, Nucl.Phys. **B173**, 365 (1980).
C. Surlykke, *On monopole suppression in lattice QED*, preprint NBI, 1994.

[66] A. Goldhaber, Hsiang–nan Li, R.R. Parwani. Phys.Rev. **D51**, 919 (1995).

[67] H.B. Nielsen, P. Olesen. Nucl.Phys. **B160**, 380 (1979).

[68] Yu.A. Simonov, Yad.Fiz. **58**, 113 (1995); A.M. Badalian, Yu.A. Simonov, Yad.Fiz. **60**, 714 (1997); A.M. Badalian, D.S. Kuzmenko, *Freezing of QCD coupling alpha(s) affects the short distance static potential*, ArXiv: hep-ph/0104097.

[69] T. Suzuki, Progr.Theor.Phys. **80**, 929 (1988); S. Maedan, T. Suzuki, Progr.Theor.Phys. **81**, 229 (1989).

[70] D.R.T. Jones, Nucl.Phys. **B75**, 531 (1974); Phys.Rev. **D25**, 581 (1982).

[71] M. Fischler, C.T. Hill, Nucl.Phys. **B193**, 53 (1981).

[72] I. Jack, H. Osborn, J.Phys. **A16**, 1101 (1983).

[73] M.E. Machacek, M.T. Vaughn, Nucl.Phys. **B222**, 83 (1983); ibid, **B249**, 70 (1985).

[74] H. Arason, D.J. Castano, B. Kesthelyi, S. Mikaelian, E.J. Piard, P. Ramond, B.D. Wright, Phys.Rev. **D46**, 3945 (1992).

[75] C. Ford, D.R.T. Jones, P.W. Stephenson, M.B. Einhorn, Nucl.Phys. **B395**, 17 (1993); C. Ford, I. Jack, D.R.T. Jones, Nucl.Phys. **B387**, 373 (1992); Erratum–ibid, **B504**, 551, (1997); ArXiv: hep-ph/0111190.

[76] O.V. Tarasov, A.A. Vladimirov, A.Yu. Zharkov, Phys.Lett. **B93**, 429 (1980); S. Larin, T. Ritberg, J. Vermaseren, Phys. Lett. **B400**, 379 (1997); M. Czakon, Nucl.Phys. **B710**, 485 (2005).

[77] J. Jersak, T. Neuhaus, H. Pfeiffer, Phys.Rev. **D60**, 054502 (1999).

[78] A.A. Abrikosov, Soviet JETP, **32**, 1442 (1957).

[79] H.B. Nielsen, P. Olesen, Nucl.Phys. **B61**, 45 (1973).

[80] E.T. Akhmedov, M.N. Chernodub, M.I. Polikarpov, M.A. Zubkov, Phys.Rev. **D53**, 2087 (1996).

[81] G. 't Hooft, Nucl.Phys. **B190**, 455 (1981).

[82] Yu.A. Simonov, Uspehi in Physics **39**, 313 (1996).

[83] M.N. Chernodub, F.V. Gubarev, JETP Lett. **62**, 100 (1995); M.N. Chernodub, F.V. Gubarev, M.I. Polikarpov, JETP Lett. **69**, 169 (1999); M.N. Chernodub, Phys.Rev. **D69**, 094504 (2004).

[84] G.S. Bali, C. Schlighter, K. Shilling, Progr.Theor.Phys.Suppl. **131**, 645 (1998).

[85] L.D. Faddeev, A.J. Niemi, Phys.Lett. **B525**, 195 (2002).

[86] L. Nottale, Int.J.Mod.Phys. **A7**, 4899 (1992); L. Nottale, *Fractal Space–Time and Microphysics*, Singapore: World Scientific, 1993; L. Nottale, in: *'Frontiers of Fundamental Physics', Proceedings of Forth International Symposium*, Hyderabad, India, 11–13 Dec., 2000; ed. B.G. Sidharth (World Scientific, Singapore, 2001).

[87] B.G. Sidharth, Int.J.Mod.Phys. **A13**, 2599 (1998).

[88] H.P. Nilles, Phys.Rep. **110**, 1 (1984).

[89] P. Langacker, N. Polonsky, Phys.Rev. **D47**, 4028 (1993); ibid, **D49**, 1454 (1994); ibid, **D52**, 3081 (1995).

[90] M.B. Green, J. Schwarz, Phys.Lett. **B149**, 117 (1984).

[91] A.G. Agnese, R. Festa, Phys.Lett. **A227**, 165 (1997).

[92] J. Schwinger, Phys.Rev. **144**, 1087 (1966); **151**, 1048; 1055 (1966); **173**, 1536 (1968); Science **165**, 757 (1969); **166**, 690 (1969).

[93] W. Deans, Nucl.Phys. **B197**, 307 (1982).

[94] C. Panagiotakopoulos, J.Phys. **A16**, 133 (1983).

[95] P.A.M. Dirac, Proc.R.Soc. (London) **A33**, 60 (1931).

[96] G. Calucci, R. Iengo, Nucl.Phys. **B223**, 501 (1983).

[97] G.J. Goebel, M.T. Thomaz, Phys.Rev. **D30**, 823 (1984).

[98] S. Coleman, in: *The Unity of the Fundamental Interactions, Eriche Lectures*, 1982, ed. A.Zichichi (Plenum, 1983), p.21.

[99] L.V. Laperashvili, H.B. Nielsen, Mod.Phys.Lett. **A14**, 2797 (1999).

[100] M. Blagojevic, P. Senjanovic, Phys.Rep. **157**, 234 (1988).

[101] D.Zwanziger, Phys.Rev. **D3**, 343 (1971).

[102] R.A. Brandt, F. Neri, D. Zwanziger, Phys.Rev. **D19**, 1153 (1979).

[103] F.V. Gubarev, M.I. Polikarpov, V.I. Zakharov, Phys.Lett. **B438**, 147 (1998).

[104] L.P. Gamberg, K.A. Milton, Phys.Rev. **D61**, 075013 (2000).

[105] L.V. Laperashvili, H.B. Nielsen, *The problem of monopoles in the Standard and Family replicated models*. A talk given by L.V.L. at the *11th Lomonosov Conference on Elementary Particle Physics*, Moscow, Russia, 21–27 Aug., 2003; ArXiv: hep-th/0311261.

[106] N.N. Bogolyubov, D.V. Shirkov, JETP, **30**, 77 (1956).

[107] S.G. Matinyan, G.K. Savvidy, Nucl.Phys. **B134**, 539 (1978).

[108] P.A. Kovalenko, L.V. Laperashvili, Phys. of Atom.Nucl. **62**, 1729 (1999).

[109] P.A. Kovalenko, L.V. Laperashvili, *The effective QCD Lagrangian and renormalization group approach*, ITEP–PH–11–97, Aug 1997, 5pp. *Talk given at 8th Lomonosov Conference on Elementary Particle Physics*, Moscow, Russia, 25–29 Aug., 1997; ArXiv: hep-ph/9711390.

[110] D.I. Olive, Phys.Rep. **49**, 165 (1979).

[111] L. Michel, L. O'Raifeartaigh, K.C. Wali, Phys.Rev. **D15**, 3641 (1977).

[112] S. Rajpoot, Nucl.Phys.Proc.Suppl. **A51**, 50 (1996).

[113] S. Dimopoulos, G. Dvali, R. Rattazzi, G.F. Guidice, Nucl.Phys. **B510**, 12 (1998).

[114] F.S. Ling, P. Ramond, Phys.Rev. **D67**, 115010 (2003).

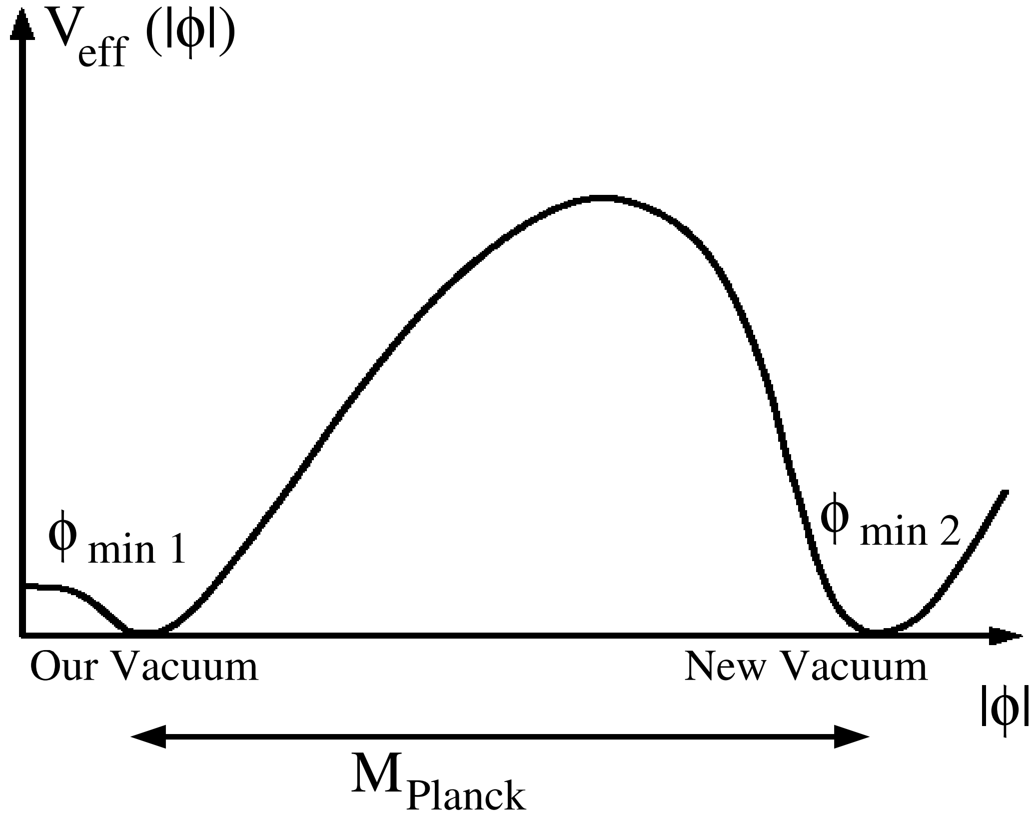

Figure 1: 2nd vacuum degenerate with usual SM vacuum. SM valid up to Planck scale except $\phi_{min2} \approx M_{\text{Planck}}$

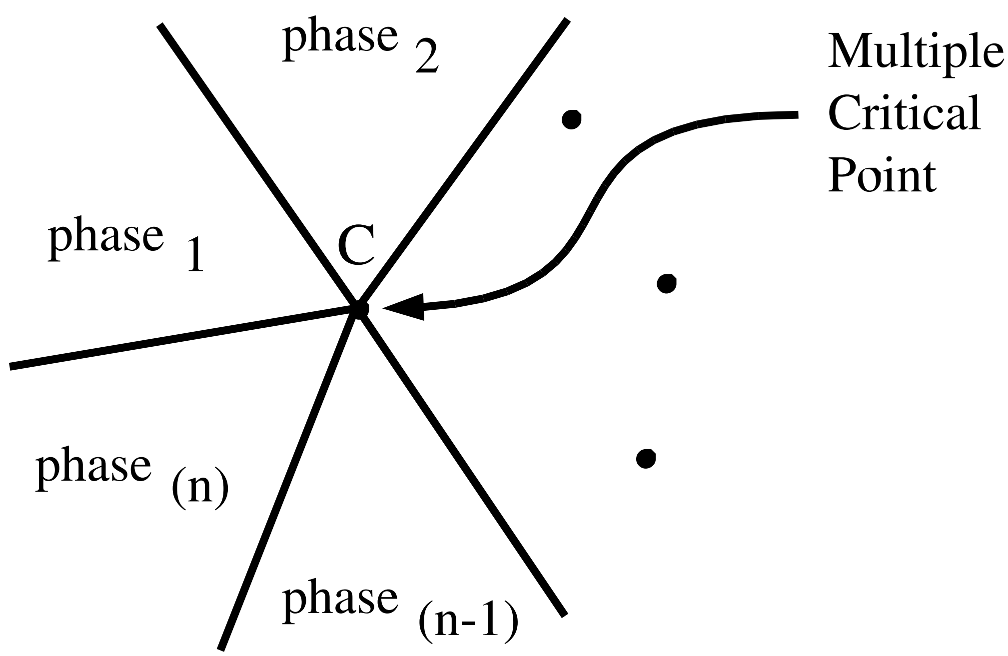

Figure 2: The schematic representation of the phase diagram of a gauge theory having n phases. The point C is the Multiple Critical Point.

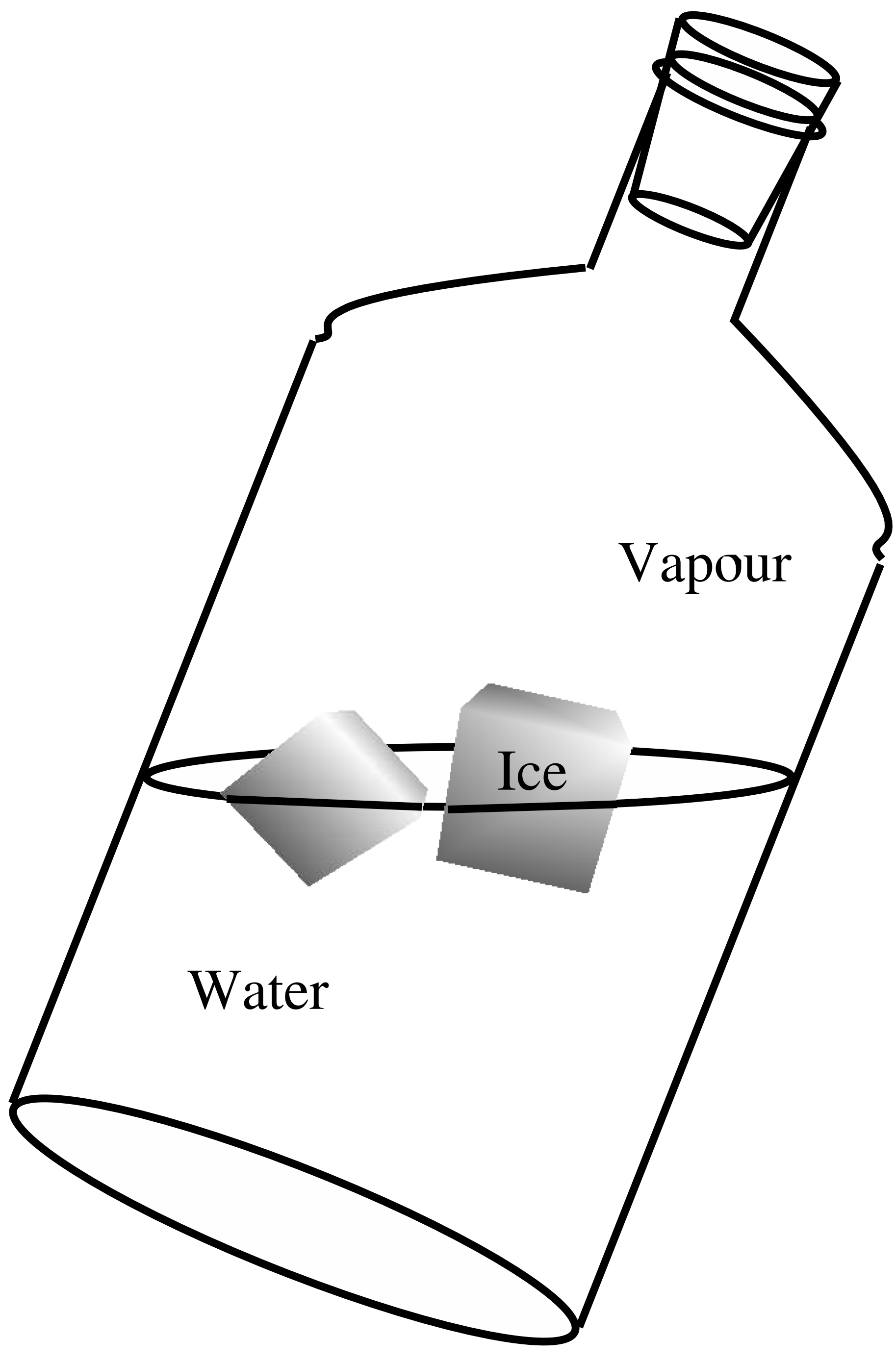

Figure 3: Triple point of water analogy: Ice, Water and Vapour. The volume, energy and a number of moles are fixed in the system.

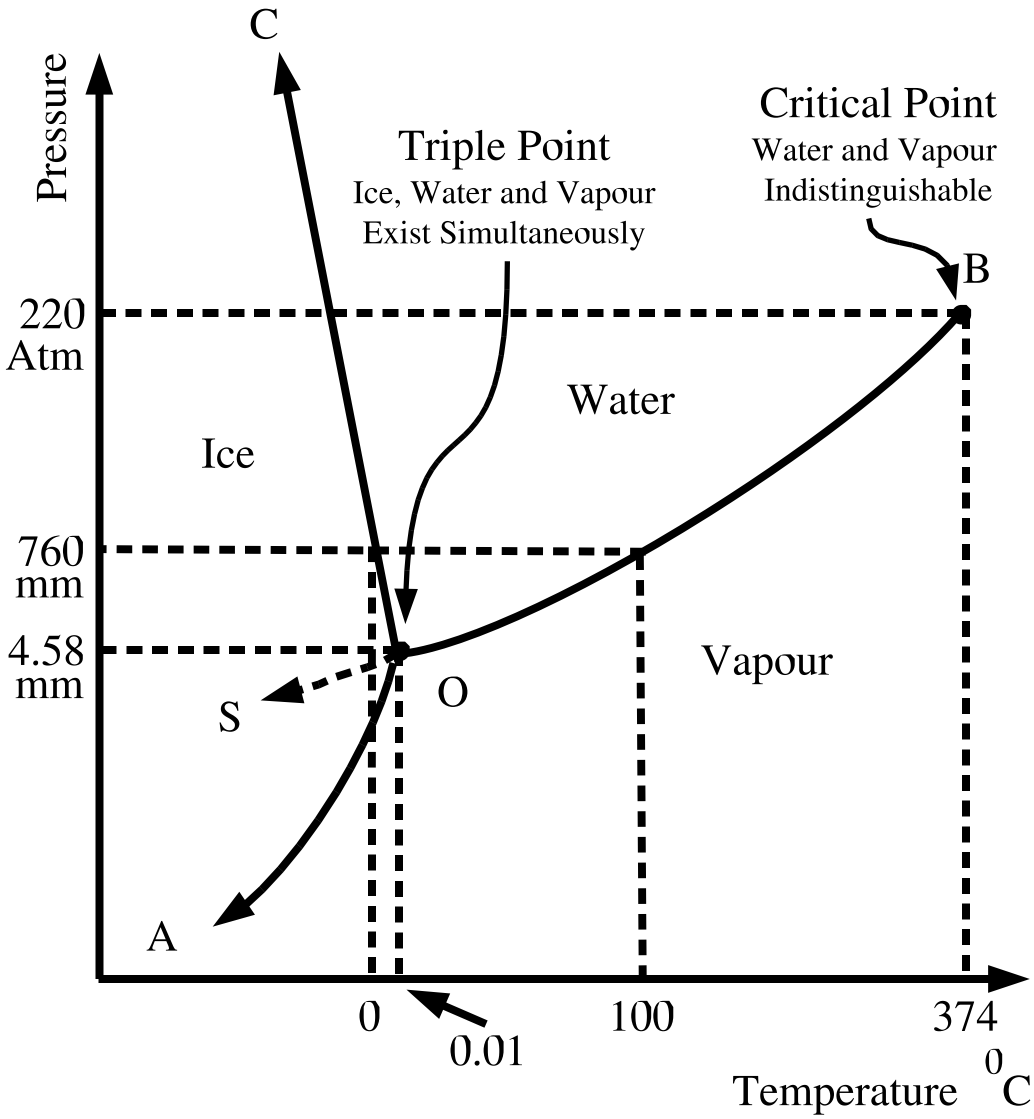

Figure 4: Triple point of water analogy. Fine–tuned intensive variables: critical temperature $T_c = 0.01\,^{\circ}\mathrm{C}$, critical pressure $P_c = 4.58$ mm Hg.

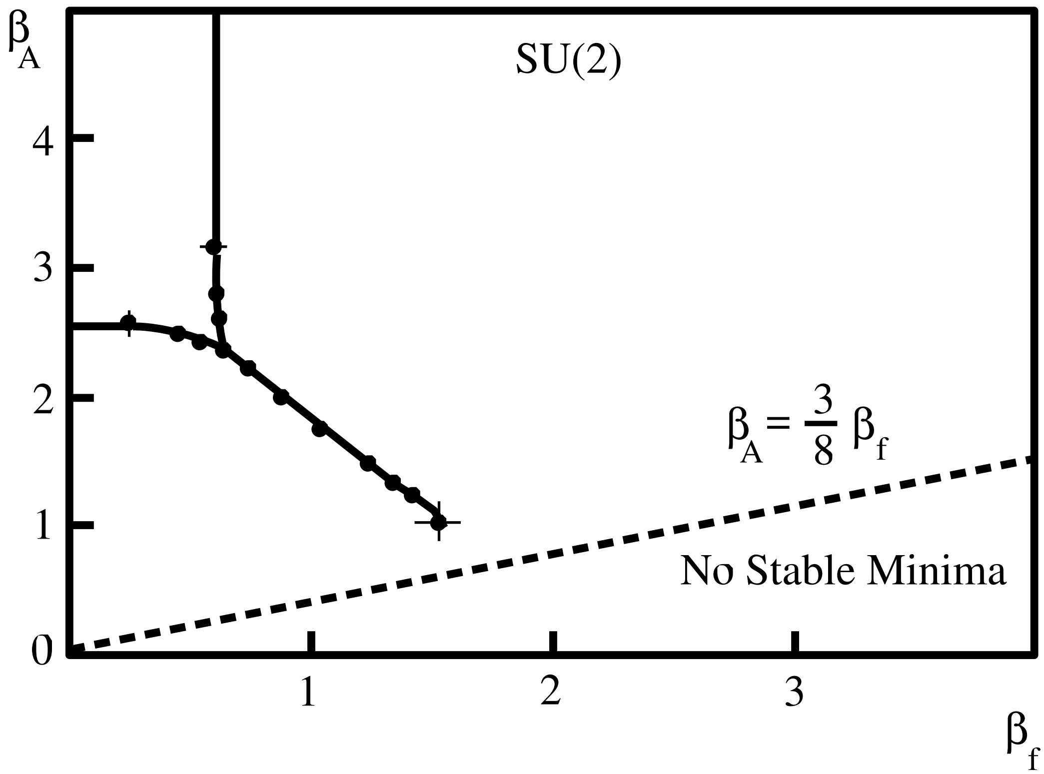

Figure 5: Phase diagram for the $SU(2)$ lattice gauge theory with the generalized Wilson lattice action. The result of Monte–Carlo simulations. Here $(\beta_{\rm f};\, \beta_{\rm A})_{crit} = (0.54;\, 2.4)$

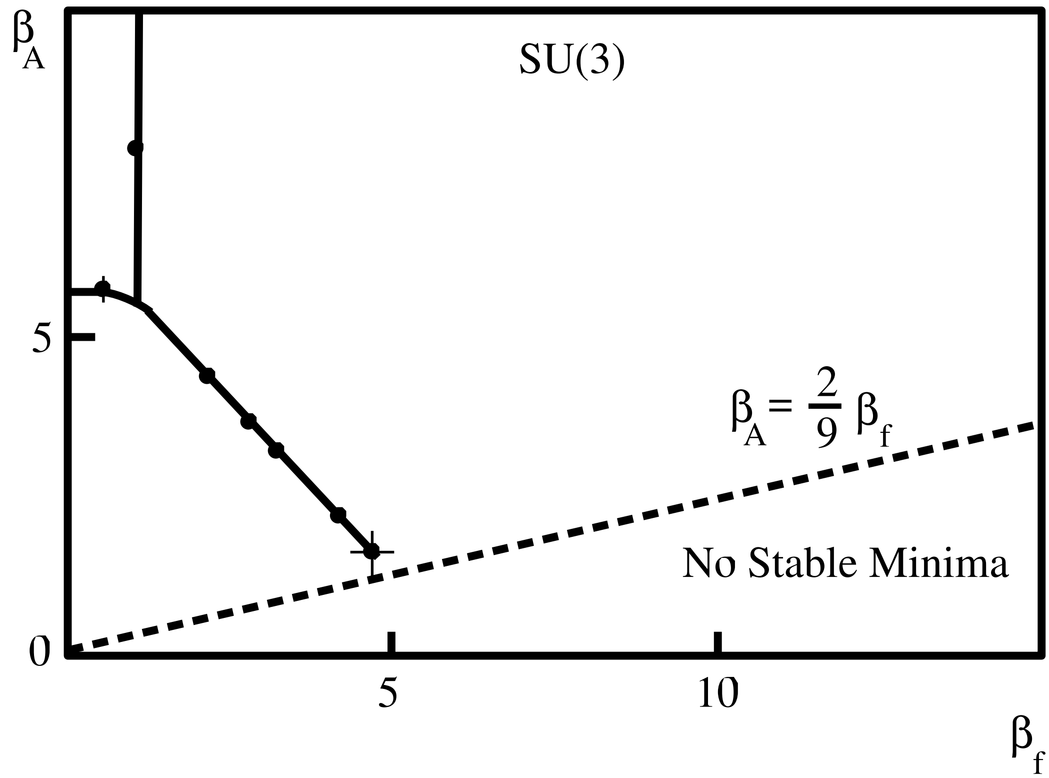

Figure 6: Phase diagram for the $SU(3)$ lattice gauge theory with the generalized Wilson lattice action. The result of Monte–Carlo simulations. Here $(\beta_{\rm f};\, \beta_{\rm A})_{crit} = (0.80;\, 5.4)$

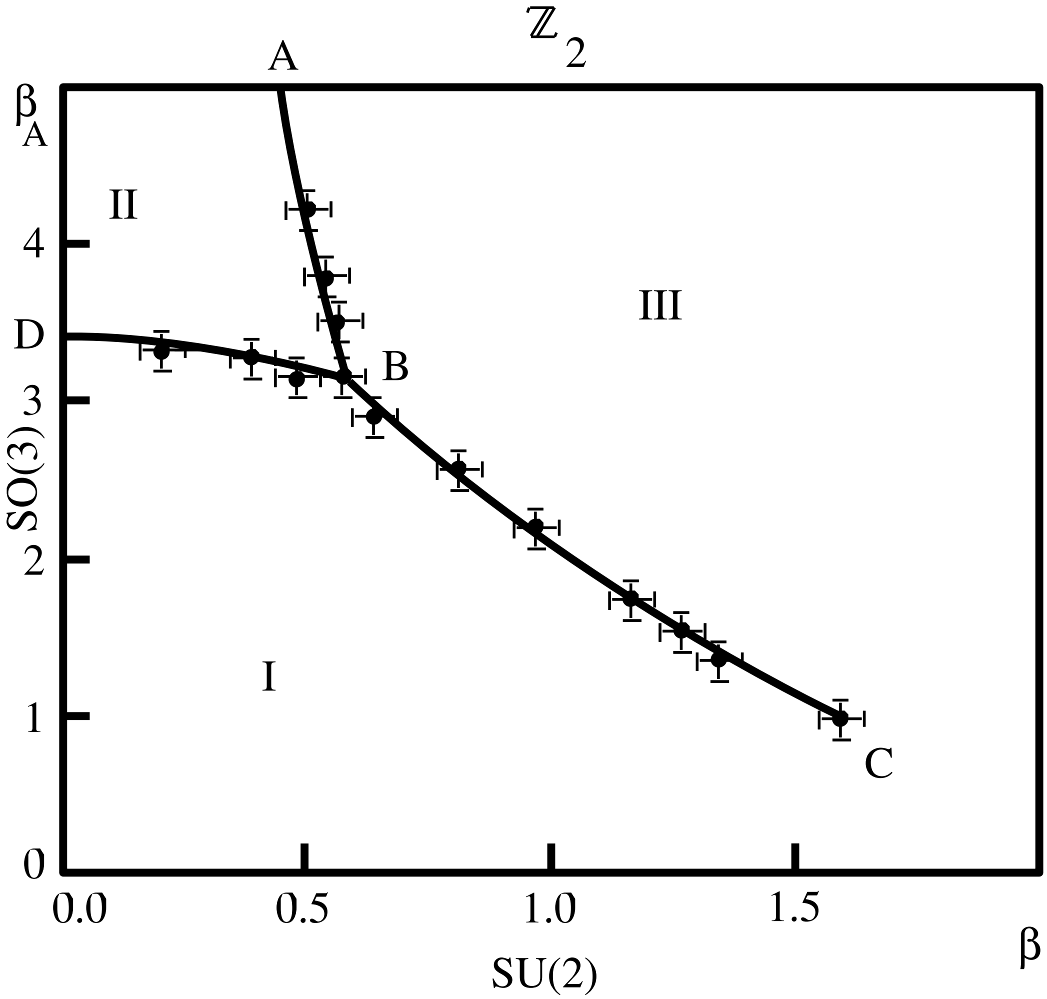

Figure 7: The phase diagram for the lattice $SU(2)$–$SO(3)$ gauge theory. The range I contains Z_2–vortices with density E and Z_2–monopoles with density M. Here $E \approx M \approx 0.5$. The range II corresponds to $E \approx 0.5, M \approx 0$, and in the range III we have $E \approx M \approx 0$.

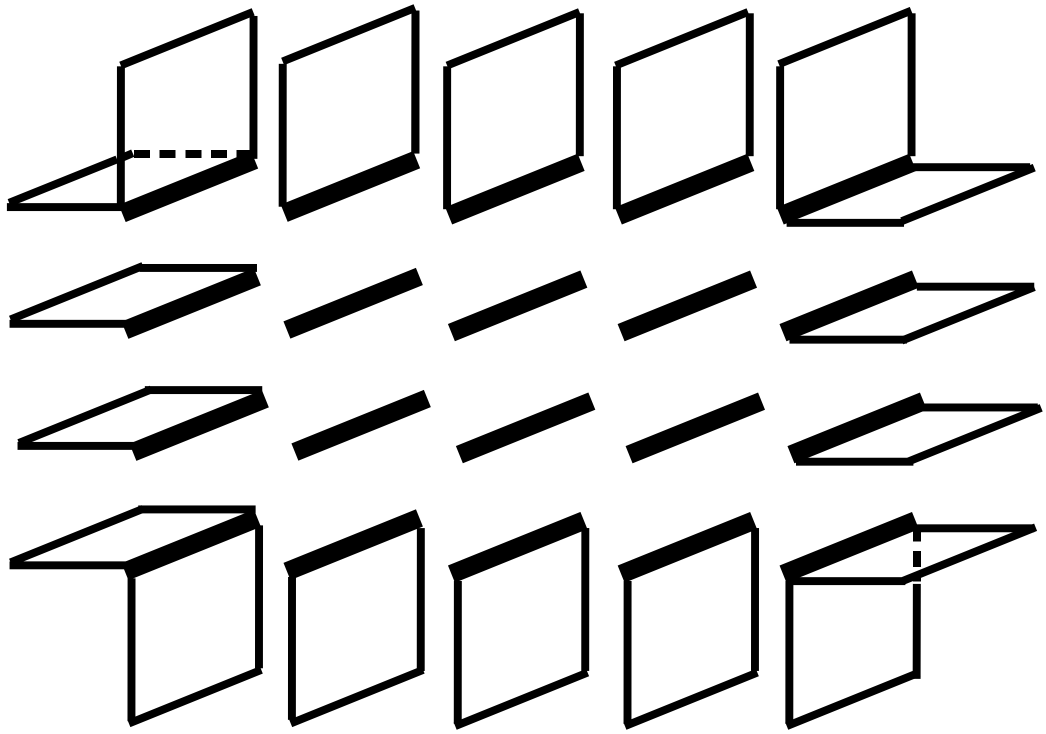

Figure 8: The closed Z_2–vortex of the 3–dimensional lattice.

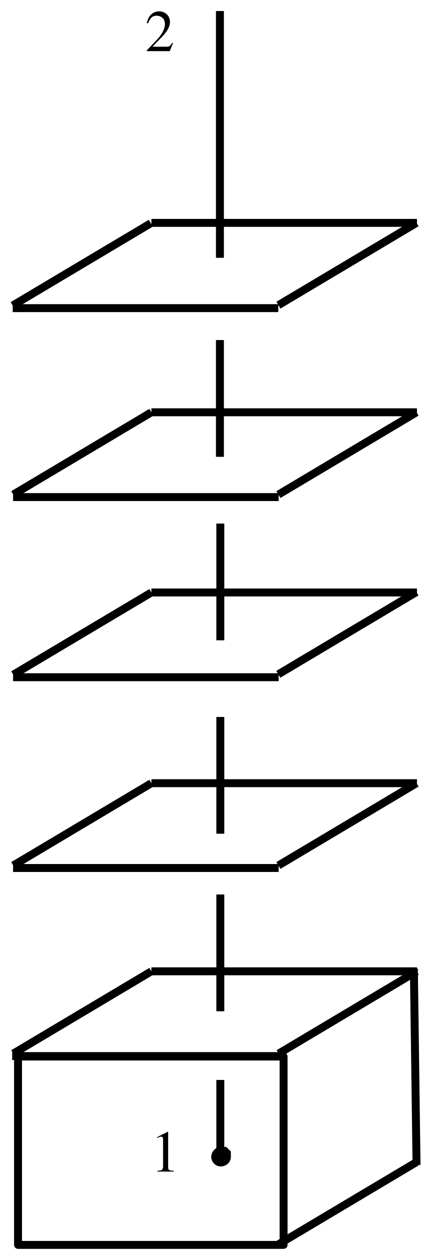

Figure 9: Z_2–monopole of the 3–dimensional lattice.

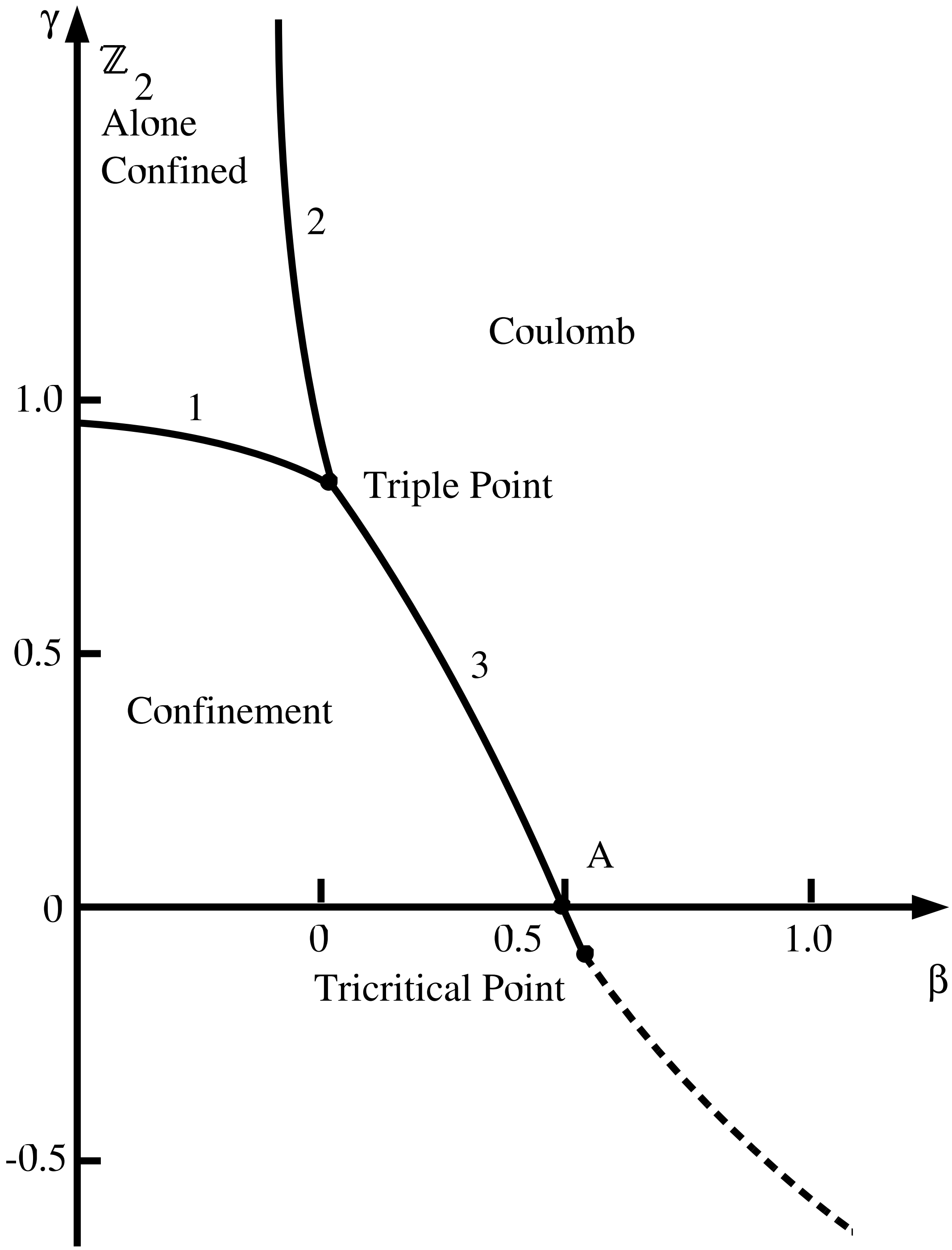

Figure 10: The phase diagram for $U(1)$ when the two–parameter lattice action is used. This type of action makes it possible to provoke the confinement Z_2 or (Z_3) alone. The diagram shows the existence of a triple (critical) point. From this triple point emanate three phase borders: the phase border "1" separates the totally confining phase from the phase where only the discrete subgroup Z_2 is confined; the phase border "2" separates the latter phase from the totally Coulomb–like phase; and the phase border "3" separates the totally confining and totally Coulomb–like phases.

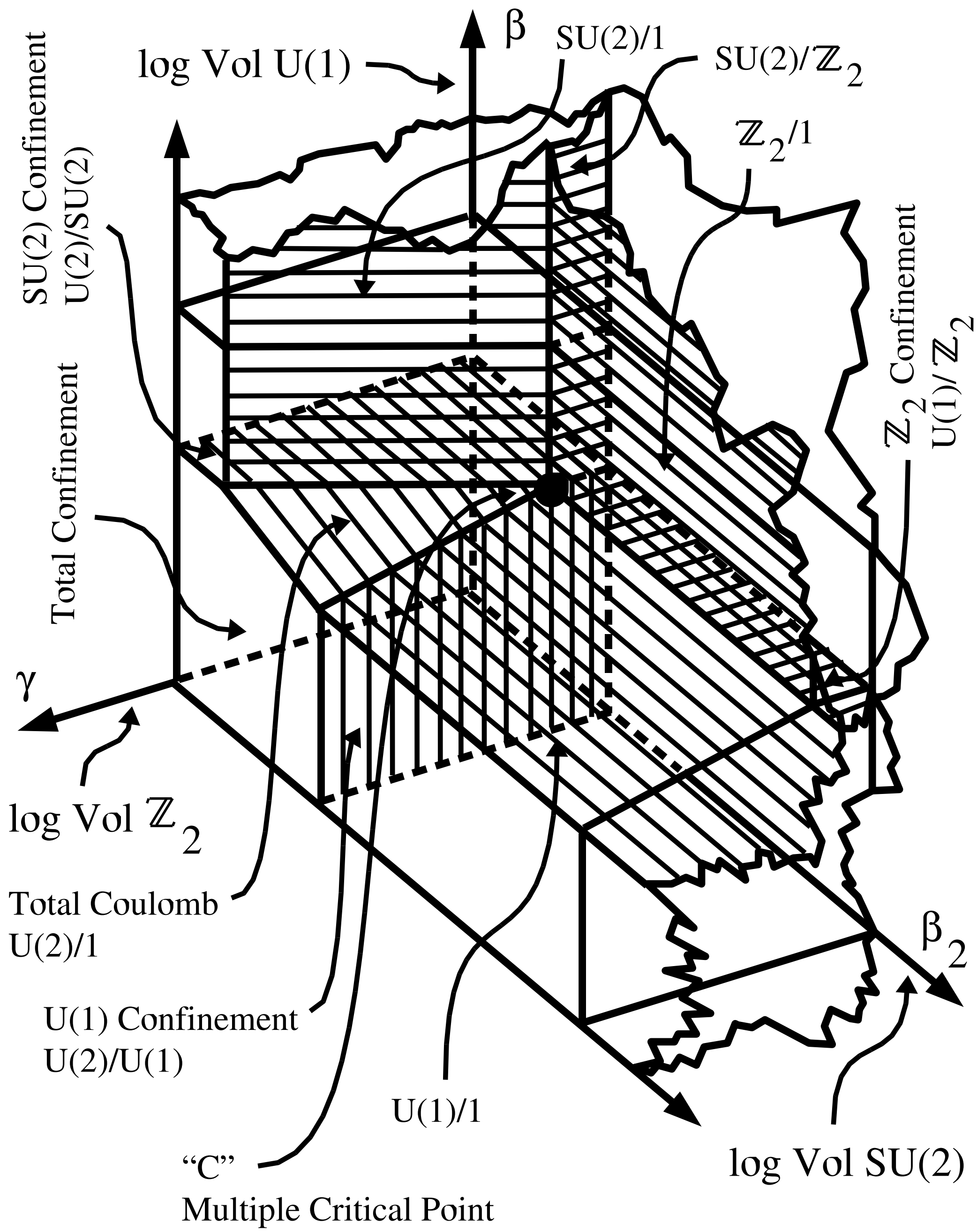

Figure 11: Phase diagram for the lattice $U(1) \times SU(2)$ gauge theory. Five phases meet at the multiple critical point.

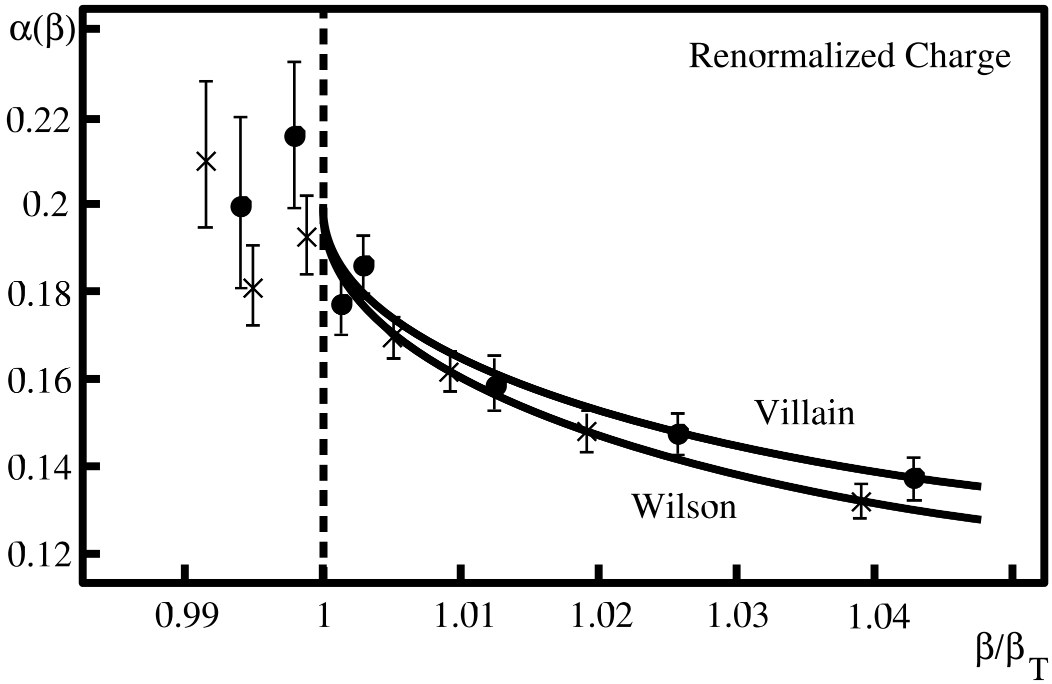

Figure 12: The renormalized electric fine structure constant plotted versus β/β_T for the Villain action (full circles) and the Wilson action (crosses). The points are obtained by the Monte–Carlo simulations method for the compact QED.

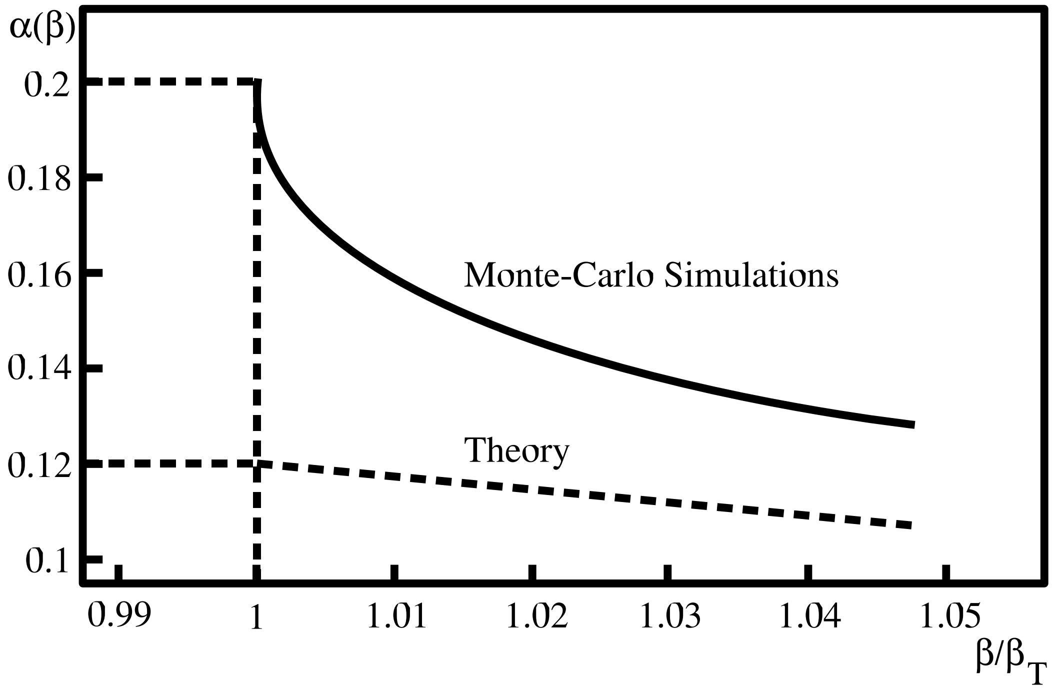

Figure 13: The behavior of the effective electric fine structure constant in the vicinity of the phase transition point obtained with the lattice Wilson action. The dashed curve corresponds to the theoretical calculations by the "Parisi improvement method".

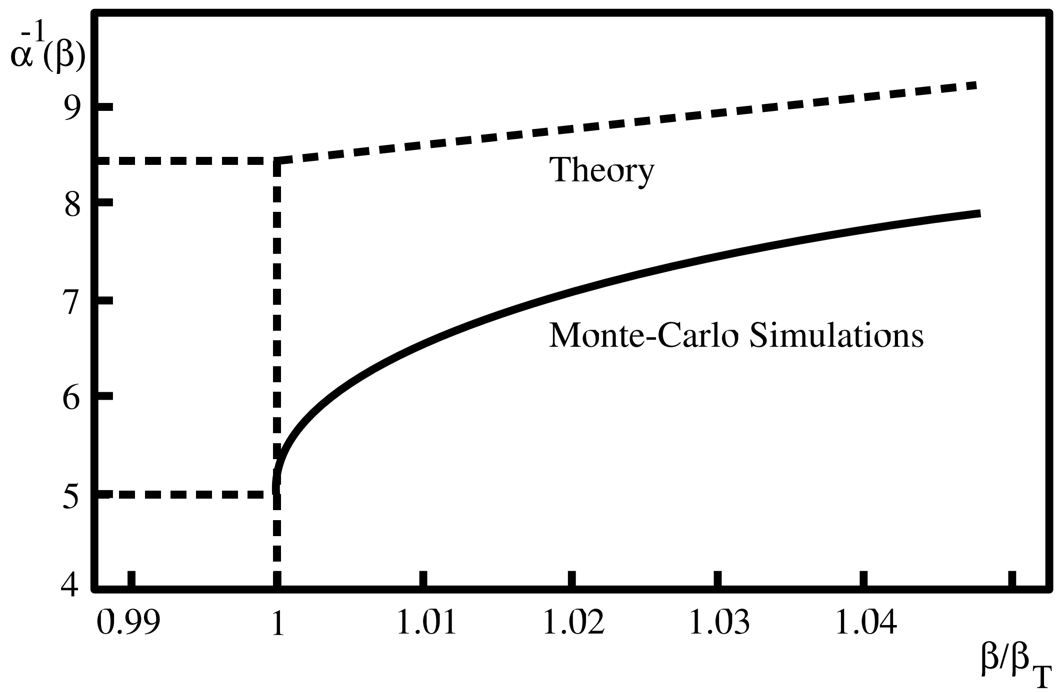

Figure 14: The behavior of the inverse effective electric fine structure constant in the vicinity of the phase transition point ploted versus β/β_T for the simple Wilson lattice action. The dashed curve corresponds to the theoretical calculations by the "Parisi improvement method".

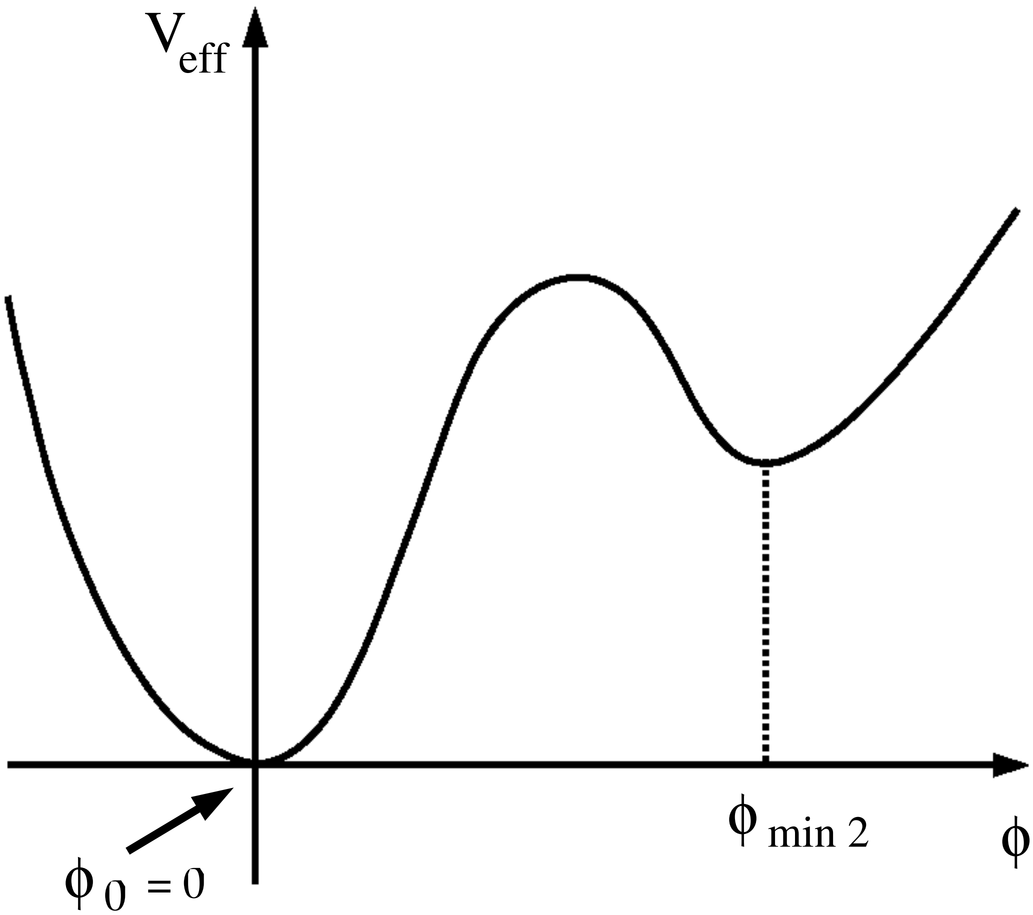

Figure 15: The effective potential $V_{eff}(\phi)$ having the first local minimum at $\phi_0 = 0$ and $V_{eff}(0) = 0$. If the second minimum occurs at $V_{eff}\,(\phi_{min2}) > 0$, this case corresponds to the "symmetric", or Coulomb–like phase.

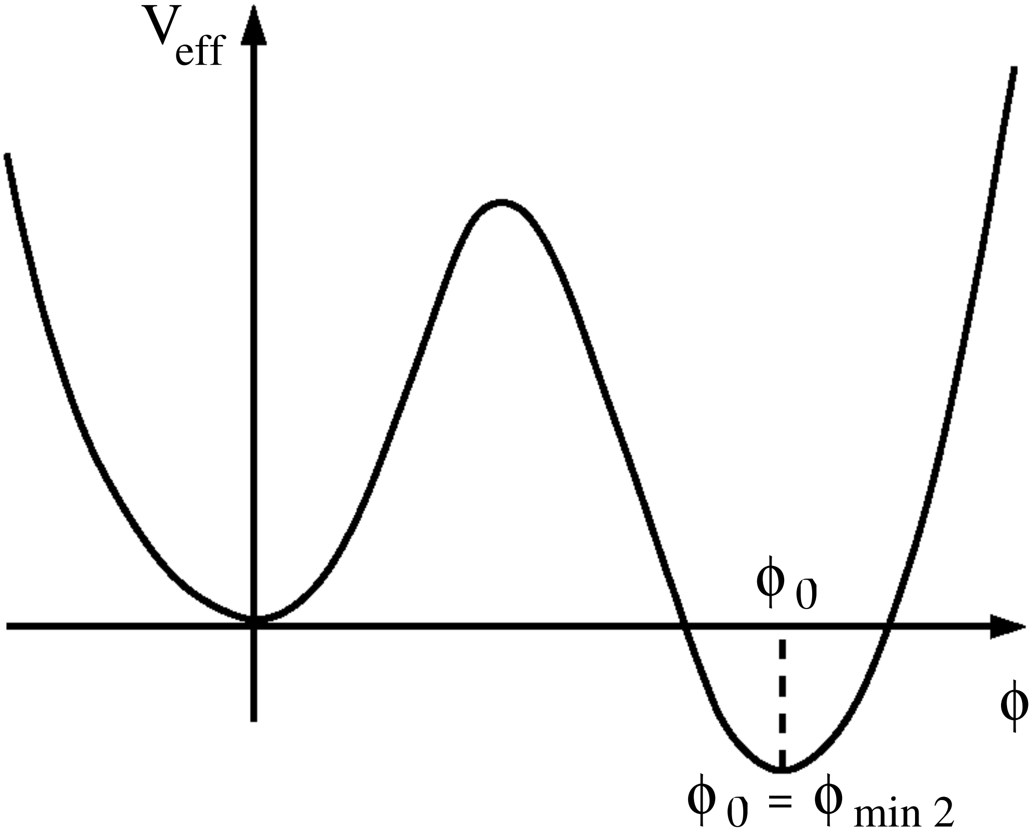

Figure 16: The effective potential $V_{eff}(\phi)$ for the confinement phase. In this case the second local minimum occurs at $\phi_0 = \phi_{min2} \neq 0$ and $V_{eff}^{min}(\phi_{min2}) < 0$.

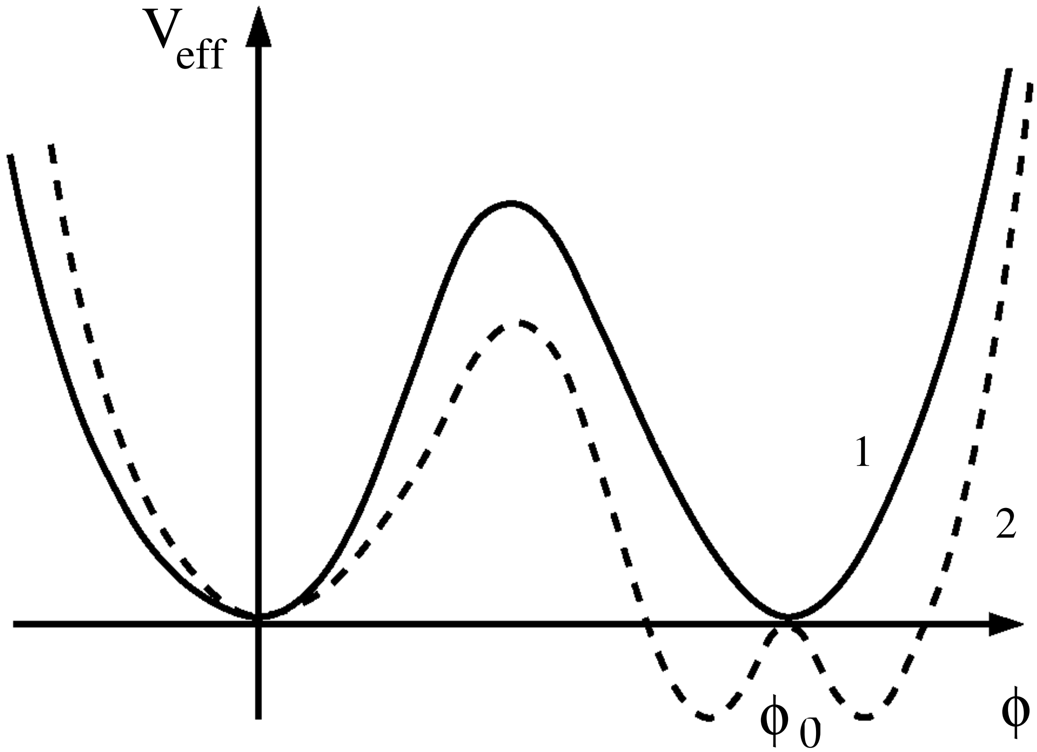

Figure 17: The effective potential V_{eff}: the curve "1" corresponds to the "Coulomb–confinement" phase transition; curve "2" describes the existence of two minima corresponding to the confinement phases.

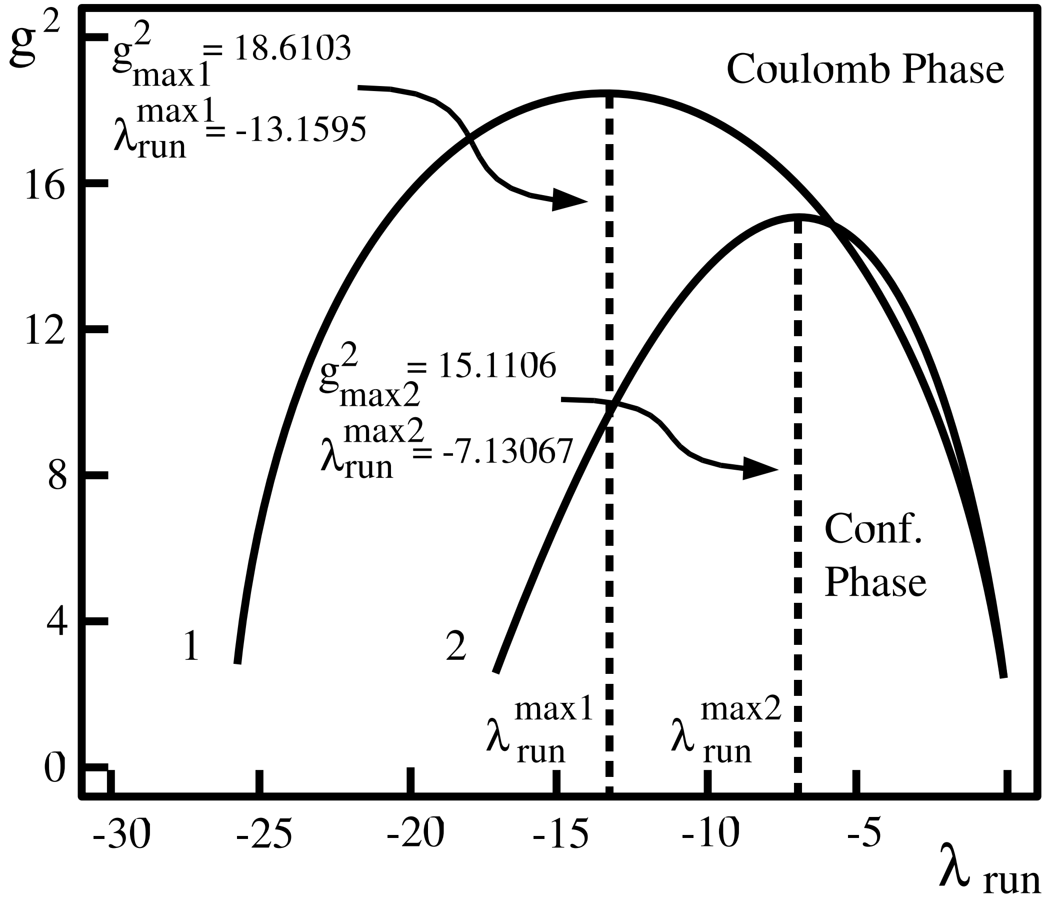

Figure 18: The one–loop (curve "1") and two–loop (curve "2") approximation phase diagram in the dual Abelian Higgs model of scalar monopoles.

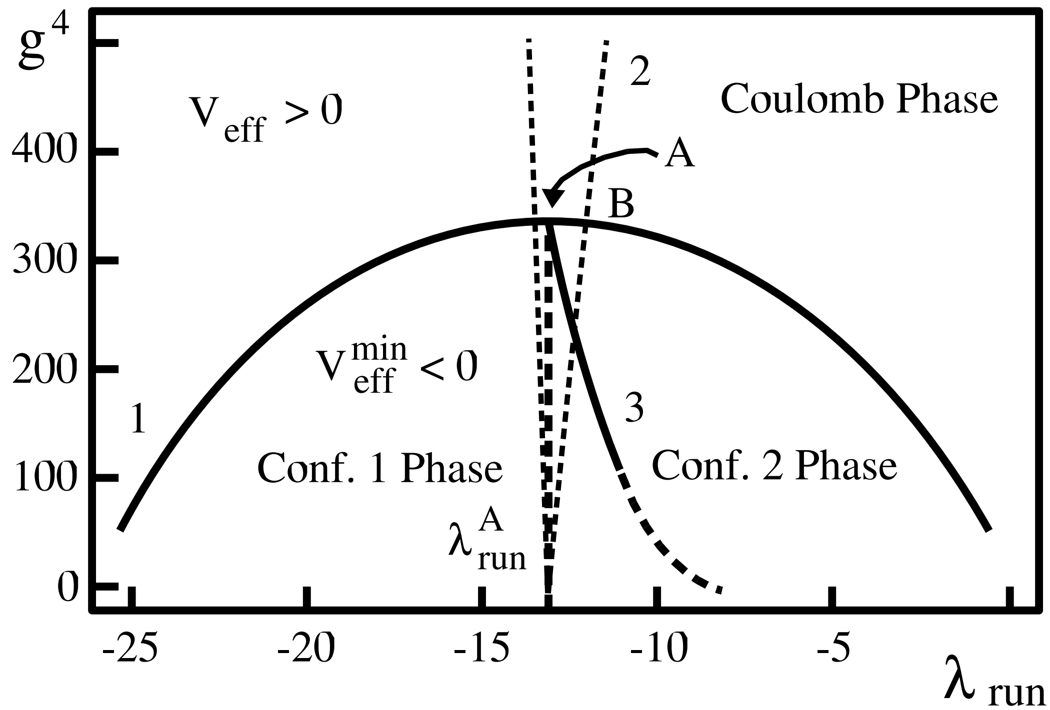

Figure 19: The phase diagram (λ_{run}; $g^4 \equiv g^4_{run}$), corresponding to the Higgs monopole model in the one–loop approximation, shows the existence of a triple point A $\left(\lambda_{(A)} \approx -13.4;\ g^2_{(A)} \approx 18.6\right)$. This triple point is a boundary point of three phase transitions: the "Coulomb–like" phase and two confinement phases ("Conf. 1" and "Conf. 2") meet together at the triple point A. The dashed curve "2" shows the requirement: $V_{eff}(\phi_0^2) = V''_{eff}(\phi_0^2) = 0$. Monopole condensation leads to the confinement of the electric charges: ANO electric vortices (with electric charges at their ends, or closed) are created in the confinement phases "Conf. 1" and "Conf. 2".

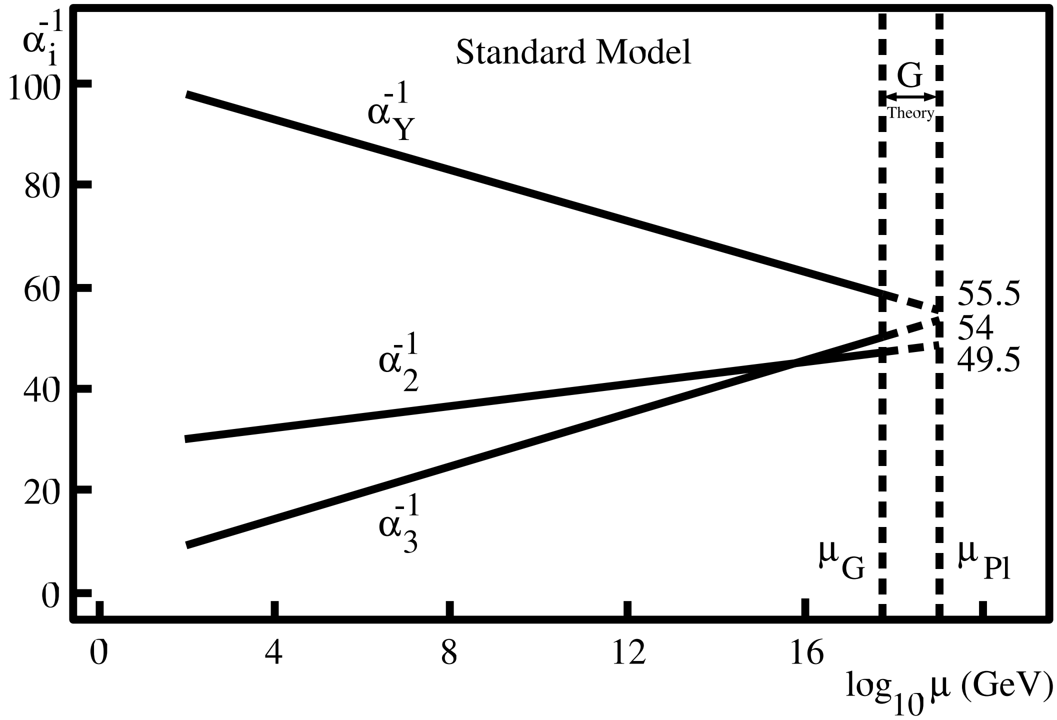

Figure 20: The evolution of three inverse running constants $\alpha_i^{-1}(\mu)$, where $i = Y, 2, 3$ correspond to $U(1)_Y$, $SU(2)$ and $SU(3)$ groups of the SM. The extrapolation of their experimental values from the Electroweak scale to the Planck scale was obtained by using the renormalization group eqations with one Higgs doublet under the assumption of a "desert". The precision of the LEP data allows to make this extrapolation with small errors. AGUT works in the region $\mu_{\rm G} \le \mu \le \mu_{\rm Pl}$.

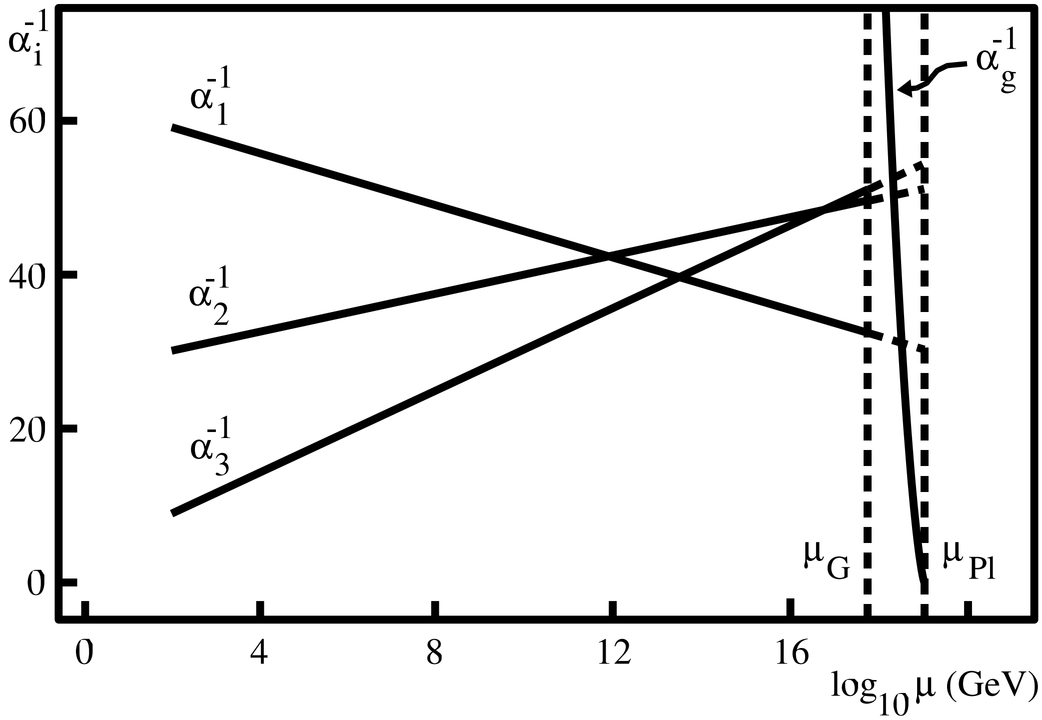

Figure 21: The intersection of the inverse "gravitational finestructure constant" $\alpha_g^{-1}(\mu)$ with $\alpha_1^{-1}(\mu)$ occurs at the point $\left(x_0, \alpha_0^{-1}\right)$: $\alpha_0^{-1} \approx 34.4$ and $x_0 \approx 18.3$, where $x = \log_{10}(\mu)$ (GeV).

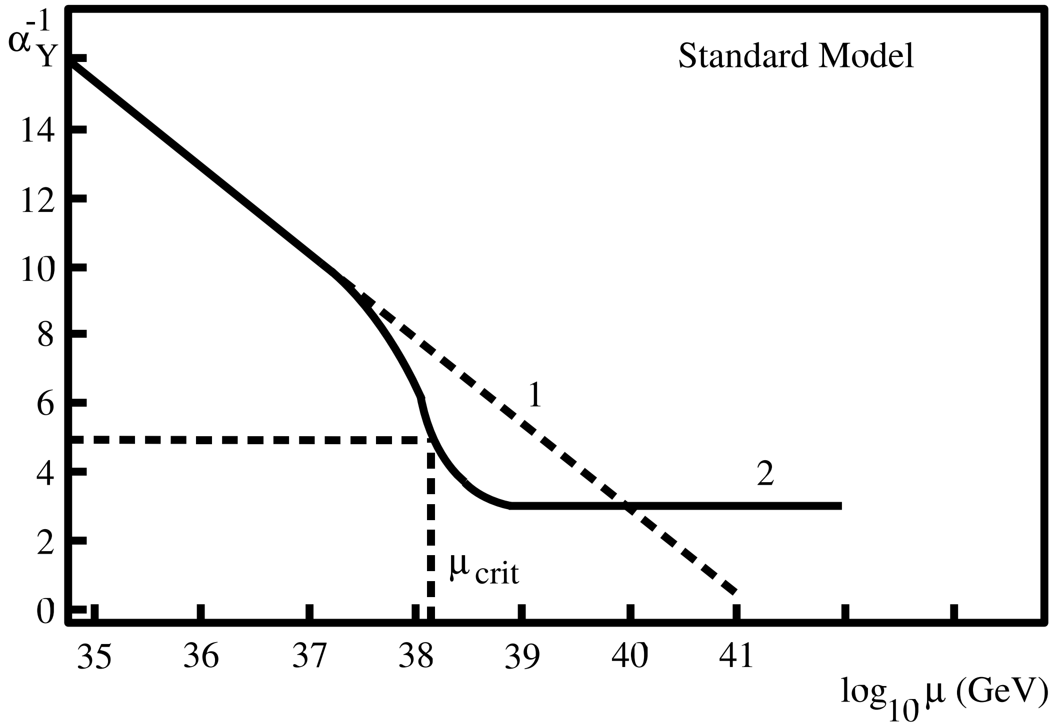

Figure 22: The evolution of the $U(1)_Y$ fine structure constant in the Standard Model with influence of monopoles at very high energies; $\mu = \mu_{crit}$ is a critical point corresponding to the phase transition "confinement–deconfinement".

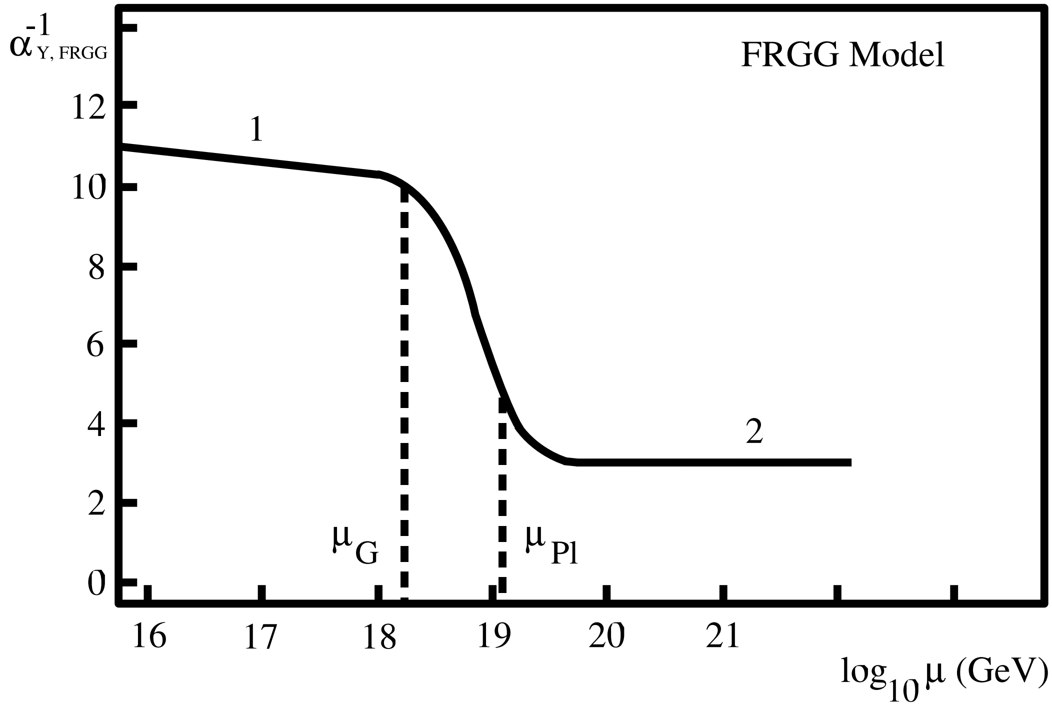

Figure 23: The evolution of the one family fine structure constant in the Family replicated gauge group model with the phase transition point at $\mu_{crit} = \mu_{Pl}$, and the FRGG symmetry breaking point at $\mu = \mu_{\rm G}$.

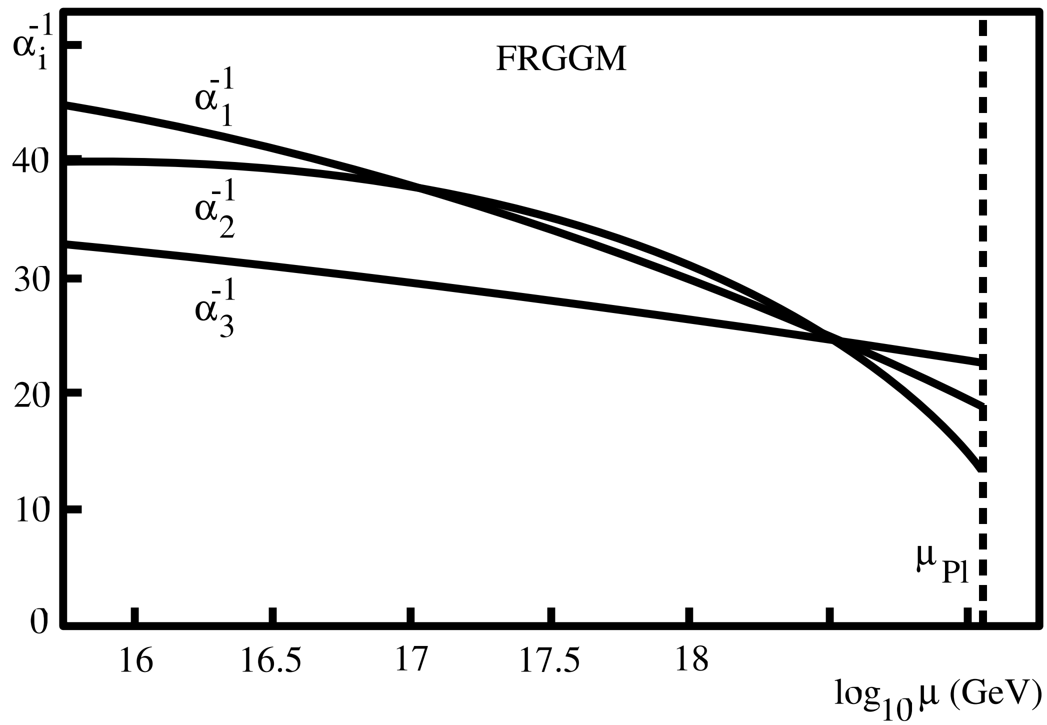

Figure 24: The evolution of fine structure constants $\alpha^{-1}_{1,\,2,\,3}(\mu)$ beyond the Standard model in the Family replicated gauge group model (FRGGM) with influence of monopoles near the Planck scale.

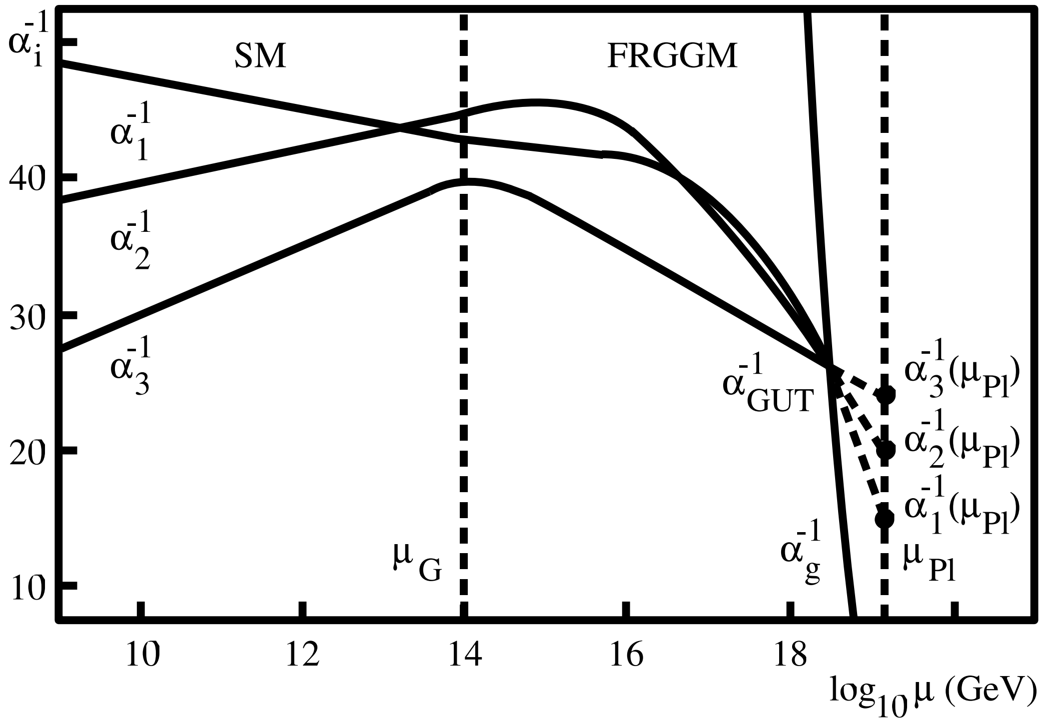

Figure 25: The evolution of $\alpha^{-1}_{1,2,3}(\mu)$ in the Standard model (SM) and beyond it. The breakdown of FRGG occurs at $\mu_G \sim 10^{14}$ GeV. It is shown the possibility of the $[SU(5)]^3$ SUSY unification of all gauge interactions, including gravity, at $\alpha^{-1}_{\rm GUT} \approx 27$ and $x_{\rm GUT} \approx 18.4$, where $x = \log_{10}(\mu)$ (GeV).

Table 1: FRGGM by Froggatt–Nielsen with the gauge group $(\mathrm{SMG})^3 \times U(1)_f$. Best fit to the conventional experimental data. All masses are running masses at 1 GeV except the top quark mass M_t which is the pole mass.

	Fitted	Experimental
m_u	3.6 MeV	4 MeV
m_d	7.0 MeV	9 MeV
m_e	0.87 MeV	0.5 MeV
m_c	1.02 GeV	1.4 GeV
m_s	400 MeV	200 MeV
m_μ	88 MeV	105 MeV
M_t	192 GeV	180 GeV
m_b	8.3 GeV	6.3 GeV
m_τ	1.27 GeV	1.78 GeV
V_{us}	0.18	0.22
V_{cb}	0.018	0.041
V_{ub}	0.0039	0.0035

Table 2: FRGGM by Froggatt–Nielsen–Takanishi with the gauge group $(\text{SMG}\times U(1)_{(B-L)})^3$. Best fit to the conventional experimental data. All masses are running masses at 1 GeV except the top quark mass which is the pole mass.

	Fitted	Experimental
m_u	4.4 MeV	4 MeV
m_d	4.3 MeV	9 MeV
m_e	1.6 MeV	0.5 MeV
m_c	0.64 GeV	1.4 GeV
m_s	295 MeV	200 MeV
m_μ	111 MeV	105 MeV
M_t	202 GeV	180 GeV
m_b	5.7 GeV	6.3 GeV
m_τ	1.46 GeV	1.78 GeV
V_{us}	0.11	0.22
V_{cb}	0.026	0.041
V_{ub}	0.0027	0.0035
$\Delta m^2_\odot$	9.0×10^{-5} eV^2	5.0×10^{-5} eV^2
Δm^2_{atm}	1.7×10^{-3} eV^2	2.5×10^{-3} eV^2
$\tan^2\theta_\odot$	0.26	0.34
$\tan^2\theta_{\text{atm}}$	0.65	1.0
$\tan^2\theta_{\text{chooz}}$	2.9×10^{-2}	$< 2.6 \times 10^{-2}$